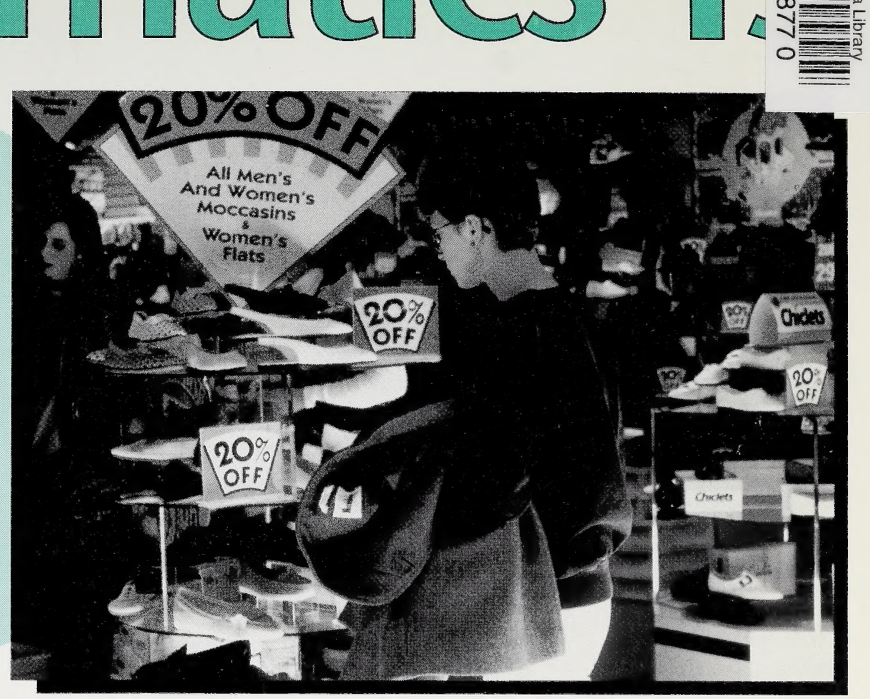
Mathematics 15



Distance Learning

Alberta EDUCATION

UNIT 1

Number Systems

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Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

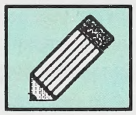
Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.

What You Already Know
- reviewing what you already know

Key Idea
- flagging important ideas

Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts

Another View
- exploring different perspectives

Computer Software
- learning by using computer software

Introduction
- introducing the unit

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape

What Lies Ahead
- previewing the unit

Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

Exploring the Topic
- actively learning new concepts

Extensions
- going on with the topic

Calculator
- using your calculator

Graphing Calculator
- using your graphing calculator

What You Have Learned
- summarizing what you have learned

Mathematics 13

Course Overview

Mathematics 13 contains 6 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Polynomials and Factoring	28%
Unit 3 Geometry	18%
Unit 4 Coordinate Geometry and Graphing	18%
Unit 5 Relations	12%
Unit 6 Statistics	14%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Number Systems

This unit covers topics dealing with number systems. Each topic contains explanations, examples, and activities to assist you in understanding number systems. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

Unit 1 Number Systems

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	Review	10
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25%	**Topic 2:** Percent	68
	• Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 2	
19%	**Topic 3:** Squares and Square Roots	91
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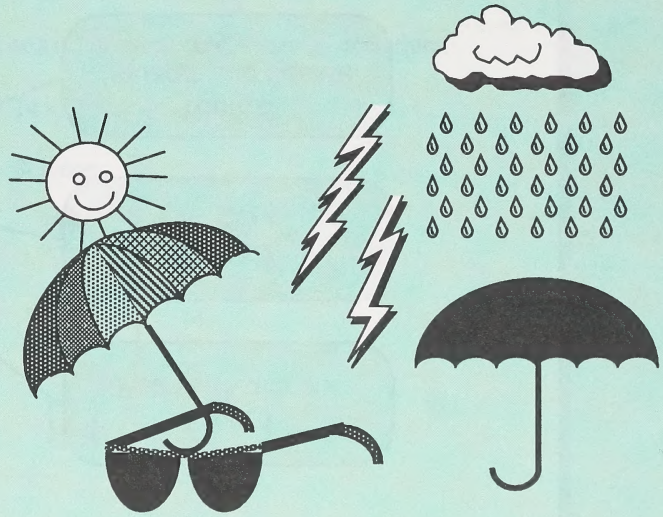
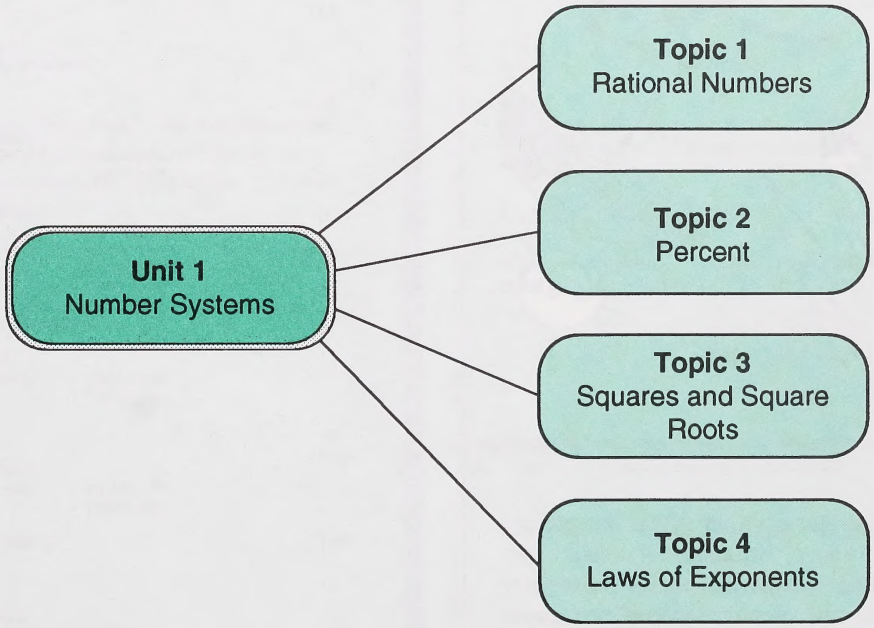
Number Systems

A weatherperson uses integers to give temperature and rational numbers to describe rainfall. Natural numbers are used for wind speed. Percentages are used to indicate the probability of precipitation for tomorrow. This unit will give means to describe the world and its events using various number systems.



Unit 1 Number Systems



What You Already Know

Recall the following:

Numbers can be grouped as natural numbers (N), whole numbers (W), or integers (I).

$N = 1, 2, 3, 4, \ldots$

$W = 0, 1, 2, 3, \ldots$

$I = \ldots -3, -2, -1, 0, 1, 2, 3, \ldots$

Example 1

Show which categories each of the following numbers belongs to. Circle the appropriate response(s).

- 5 N W I

  Solution:

  5 (N) (W) (I)

- -12 N W I

  Solution:

  -12 N W (I)

- 0 N W I

  Solution:

  0 N (W) (I)

- 229 N W I

  Solution:

  229 (N) (W) (I)

Example 2

Use a calculator for the following operations.

- 14×27

  Solution:

Enter	Display
14	14
×	14
27	27
=	378

- $3198 \div 78$

Solution:

Enter	Display
3198	3198
+	3198
78	78
=	41

- $365 - 157$

Solution:

Enter	Display
365	365
−	365
157	157
=	208

- $36 + 144$

Solution:

Enter	Display
36	36
+	36
144	144
=	180

Although calculators are used to a great extent for operation with numbers, you should be able to do simple operations with whole numbers using paper and pencil techniques.

Example 3

Do the following operations.

- $46 + 3401 + 244$

Solution:

$$\begin{array}{r} 46 \\ 3401 \\ +\ 244 \\ \hline 3961 \end{array}$$

- $3647 - 756$

Solution:

$$\begin{array}{r} 3647 \\ -\ 756 \\ \hline 2891 \end{array}$$

- 26×42

Solution:

$$\begin{array}{r} 26 \\ \times\ 42 \\ \hline 52 \\ 1040 \\ \hline 1092 \end{array}$$

- $6240 \div 13$

Solution:

$$\begin{array}{r} 480 \\ 13\overline{)6240} \\ \underline{52} \\ 104 \\ \underline{104} \\ 00 \\ \underline{0} \\ 0 \end{array}$$

Example 4

- Find the L.C.M. for 2, 3, and 4 using the product of prime numbers (Method 1).

Solution:

$2 = 2$

$3 = (3)$

$4 = (2 \times 2)$

$\text{L.C.M.} = 2 \times 2 \times 3$

$= 12$

- Find the L.C.M. for 2, 3, and 4 dividing out common factors (Method 2).

Solution:

$$\begin{array}{r|lll} 2 & 2 & 3 & 4 \\ \hline & 1 & 3 & 2 \end{array}$$

$\text{L.C.M.} = 2 \times 1 \times 3 \times 2$

$= 12$

Note: Prime numbers are numbers that have exactly two factors. These factors are one and the number itself.

Note: 2 does not divide evenly into 3; thus, write the 3 below the line.

- Find the L.C.M. for 6 and 15 using Method 1.

Solution:

$6 = (2) \times (3)$
$15 = 3 \times (5)$

$L.C.M. = 2 \times 3 \times 5$
$= 30$

- Find the L.C.M. for 6 and 15 using Method 2.

Solution:

3	6	15
	2	5

$L.C.M. = 3 \times 2 \times 5$
$= 30$

- Find the L.C.M. for 54 and 243 using Method 1.

Solution:

$54 = (2) \times 3 \times 3 \times 3$
$243 = (3 \times 3 \times 3 \times 3 \times 3)$

$L.C.M. = 2 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 486$

- Find the L.C.M. for 54 and 243 using Method 2.

Solution

3	54	243
9	18	81
	2	9

$L.C.M. = 3 \times 9 \times 2 \times 9$
$= 486$

Example 5

Method 1

- Find the greatest common factor for 25 and 15 by prime factorization.

Solution:

$25 = (1) \times 5 \times (5)$
$15 = (1) \times 3 \times (5)$

$G.C.F. = 1 \times 5$
$= 5$

Note that the prime factors of 25 are 1 and 5 and $1 \times 5 \times 5 = 25$.
The prime factors of 15 are 1, 3, and 5 and $1 \times 3 \times 5 = 15$.

Method 2

- Find the greatest common factor for 25 and 15 by dividing out common factors.

Solution:

$$\begin{array}{r|ll} 5 & 25 & 15 \\ \hline & 5 & 3 \end{array}$$

G.C.F. = 5

A common factor of both 25 and 15 is 5.

Note that the L.C.M. of 25 and 15 would be $5 \times 5 \times 3 = 75$.
The G.C.F. of 25 and 15 is 5.

- Find the G.C.F. for 7 and 13 using Method 1 and Method 2.

Method 1

Solution:

$$7 = \boxed{1} \times 7$$
$$13 = \boxed{1} \times 13$$

G.C.F. = 1

Method 2

Solution:

$$\begin{array}{r|ll} 1 & 7 & 13 \\ \hline & 7 & 13 \end{array}$$

Since only 1 divides into both 7 and 13, the G.C.F. is 1.

- Find the G.C.F. for 24, 18, and 30 using Method 1 and Method 2.

Method 1

Solution:

$$24 = \boxed{1} \times \boxed{2} \times 2 \times 2 \times \boxed{3}$$
$$18 = \boxed{1} \times \boxed{2} \times \boxed{3} \times 3$$
$$30 = \boxed{1} \times \boxed{2} \times \boxed{3} \times 5$$

G.C.F. $= 1 \times 2 \times 3$
$= 6$

Method 2

Solution:

$$\begin{array}{r|lll} 3 & 24 & 18 & 30 \\ 2 & 8 & 6 & 10 \\ \hline & 4 & 3 & 5 \end{array}$$

G.C.F. $= 3 \times 2$
$= 6$

Note: All of 24, 18, and 30 **must** be divided evenly by the common factors.

Now that you have looked at material you have studied previously, go to the **Review** to apply your understanding of this material.

Review

1. Show the categories to which each of the following numbers belongs by circling the appropriate letter.

   a. -502 *N* *W* *I*

   b. 37 *N* *W* *I*

   c. 1 *N* *W* *I*

   d. 0 *N* *W* *I*

2. Use a calculator to complete the following.

   a. 139×520

   b. $26\,124 - 11\,235$

   c. $5280 \div 176$

   d. $340 + 1129 + 2670$

3. Do the following using paper and pencil techniques.

   a. $87 + 163 + 1314 + 2991$

   b. $97\,500 - 87\,502$

   c. 126×48

   d. $516 \div 12$

4. Find the L.C.M. for the following.

   a. 2, 14, 49

   b. 18, 30

5. Find the G.C.F. for the following.

   a. 36, 60

   b. 14, 49, 98

Now go to the **Review** solutions in the **Appendix**.

If you found this review somewhat difficult, you may need to look back at **Mathematics 9, Unit 2** called Number Systems and Operations.

Topic 1 Rational Numbers

Introduction

Historically people are placed into various categories whether they like it or not. You speak of short or tall people, blond or brunette people, or boring or exciting people.

Numbers fit into certain categories as well. In the past you have learned about some categories such as whole numbers, natural numbers and integers. In this topic, you will learn about the category called rational numbers.

What Lies Ahead

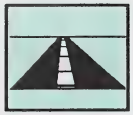
Throughout the topic you will learn to

1. understand the relationship between integers and rational numbers
2. perform addition and subtraction involving fractions
3. perform multiplication and division involving fractions
4. perform addition and subtraction involving decimals
5. perform multiplication and division of decimals
6. round and estimate with decimals
7. change fractions to decimals
8. change decimals to fractions
9. perform calculations using the order of operations
10. perform calculations involving integers
11. perform calculations involving rational numbers

Now that you know what to expect, turn the page to begin your study of rational numbers.

Exploring Topic 1

Activity 1



Understand the relationship between integers and rational numbers.

Rational numbers are numbers which can be changed to fractional number form. All integers are rational numbers, but not all rational numbers are integers. This is like saying all girls are people, but not all people are girls.

Consider the integer 5. This integer can be written as $\frac{5}{1}$ because $5 = \frac{5}{1}$. This is true for all integers.

You can go even one step further. We know that $\frac{10}{2} = 5$ and 5 is an integer. Since we are able to write 5 as $\frac{10}{2}$ (this is a true statement), then 5 is also a rational number. Thus, 5 is both an integer and a rational number.

The reverse is not always true.

For example, we cannot write $\frac{2}{3}$ or $\frac{2}{5}$ as integers. There are many more such rational numbers that are not integers.

You can conclude that **all integers are rational numbers**, but only **some rational numbers are integers**.

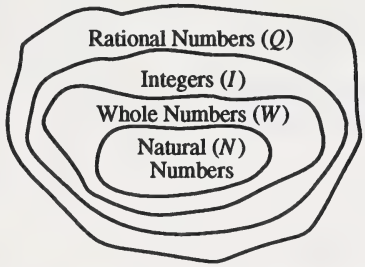
Integers are made up of the natural numbers, their negative counterparts, and zero. This may be represented as $I = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

Rational numbers are numbers that can be written in a fractional number form. Rationals include positive and negative integers, positive and negative fractions, positive and negative decimals, and zero.

Some examples of rational numbers are as follows:

$$Q = -6\frac{2}{3}, -\frac{15}{3}, -\frac{2}{7}, -\frac{0}{6}, \frac{3}{9}, \frac{3}{4}, \frac{21}{4}, 0, 4, 2, 6, 5, \frac{1}{4}$$

The Venn diagram shows the order of the categories of numbers and how the different categories are related.



You will notice that most of the properties of integers also apply to rational numbers.

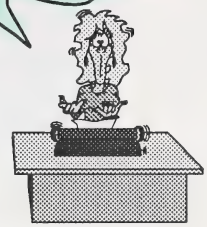
For example, an integer multiplied by an integer gives an integer and a rational number multiplied by a rational number gives a rational number.

Is this pattern true for all other operations? No, a rational number divided by a rational number gives a rational number, but an integer divided by an integer does not necessarily give an integer. Take integer 2 divided by integer 3. The result, $\frac{2}{3}$, is a rational number not an integer.

Fractional number form means in fraction form such as $\frac{2}{3}$.

The upper case Q is used to represent rational numbers.

The Venn diagram on this page shows that every natural number is also a whole number, that every whole number is also an integer, and that every integer is a rational number.



Remember that all integers are rational numbers but not all rational numbers are integers.

Look at a few examples.

Example 1

Show that the following numbers are **not** integers.

- $\frac{22}{7}$

  Solution:

  $$\frac{22}{7} = 3\frac{1}{7}$$

- $\frac{-17}{4}$

  Solution:

  $$\frac{-17}{4} = -4\frac{1}{4}$$

- $\frac{11}{9}$

  Solution:

  $$\frac{11}{9} = 1\frac{2}{9}$$

- $\frac{-102}{4}$

  Solution:

  $$\frac{-102}{4} = -25\frac{2}{4}$$
  $$= -25\frac{1}{2}$$

These numbers are all mixed number. Since they have a fraction part, they are not integers.

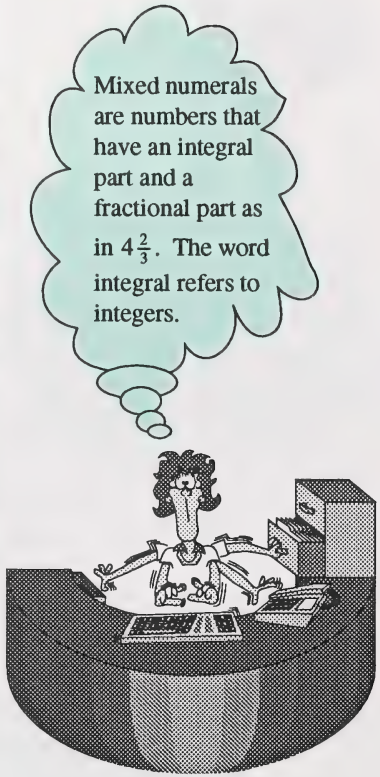
Example 2

By simplifying as much as possible tell which of the following are integers.

- $\frac{-28}{1}$

  Solution:

  $$\frac{-28}{1} = -28 \quad \text{(integer)}$$


Mixed numerals are numbers that have an integral part and a fractional part as in $4\frac{2}{3}$. The word integral refers to integers.

- $\frac{21}{4}$

Solution:

$\frac{21}{4} = 5\frac{1}{4}$ (not an integer)

- $\frac{35}{3}$

Solution:

$\frac{35}{3} = 11\frac{2}{3}$ (not an integer)

- $\frac{18}{3}$

Solution:

$\frac{18}{3} = 6$ (integer)

- $\frac{-39}{13}$

Solution:

$\frac{-39}{13} = -3$ (integer)

- $\frac{49}{-14}$

Solution:

$\frac{49}{-14} = -3\frac{7}{14}$

$= -3\frac{1}{2}$ (not an integer)

- $\frac{52}{12}$

Solution:

$\frac{52}{12} = 4\frac{4}{12}$ (not an integer)

$= 4\frac{1}{3}$

Do any three of the following five questions.

1. Show that the following are integers.

   a. $\frac{27}{3}$ b. $\frac{-31}{1}$

   c. $\frac{36}{18}$ d. $\frac{-42}{7}$

   e. $\frac{-35}{7}$ f. $\frac{-42}{14}$

   g. $\frac{51}{17}$ h. $\frac{-54}{3}$

2. Write the following rational numbers as integers.

   a. $\frac{28}{7}$ b. $\frac{26}{13}$

   c. $\frac{39}{3}$ d. $\frac{52}{4}$

   e. $\frac{51}{17}$ f. $\frac{57}{19}$

   g. $\frac{-60}{4}$ h. $\frac{24}{-3}$

3. All integers are rational numbers but not all rational numbers are integers. Explain and give examples to show this.

4. Show that the following numbers are rational numbers.

   a. 25 b. $-1\frac{3}{4}$

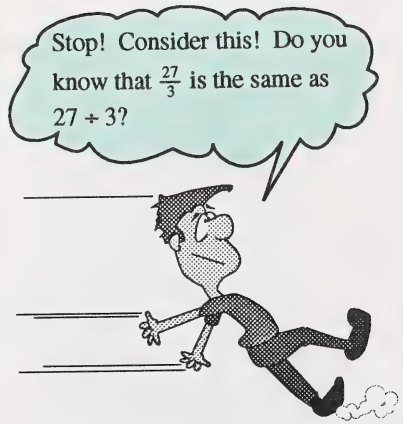
   c. $7\frac{1}{3}$ d. -31

   e. $1\frac{9}{7}$ f. 34

   g. $3\frac{1}{7}$ h. $\frac{36}{48}$

5. Can you write a list of integers that are not rational numbers? If not, why not?

For solutions to **Activity 1**, turn to the **Appendix**, **Topic 1**.



Activity 2

Perform addition and subtraction involving fractions.

An interesting relationship exists between equivalent fractions.

Look at the fractions $\frac{1}{5}$ and $\frac{3}{15}$. Note first that they are rational numbers. Then observe that $\frac{1}{5}$ is a basic fraction and $\frac{3}{15}$ is not. If you divide the numerator and the denominator of $\frac{3}{15}$ by 3, you get $\frac{1}{5}$.

You can now say that $\frac{1}{5}=\frac{3}{15}$ or $\frac{1}{5}$ and $\frac{3}{15}$ are equivalent fractions.

You need to understand equivalent fractions in order to add and subtract fractions.

Equivalent fractions are those fractions that can be reduced to the same basic fraction. For example $\frac{5}{15}$ and $\frac{3}{9}$ are equivalent fractions because $\frac{5}{15}=\frac{1}{3}$ and $\frac{3}{9}=\frac{1}{3}$.

The following graphic shows how equivalent fractions can be shown in picture form.

Consider the fractions $\frac{3}{12}$ and $\frac{5}{20}$. These are equivalent fractions since $\frac{3+3}{12+3}=\frac{1}{4}$ and $\frac{5+5}{20+5}=\frac{1}{4}$.

They reduce or simplify to the same basic fraction namely $\frac{1}{4}$.

A basic fraction is a fraction that cannot be reduced to a simpler form. Some examples are $\frac{2}{3}$, $\frac{1}{5}$, $\frac{1}{2}$, and $\frac{3}{4}$. Can you think of any number other than 1 that will divide into both the numerator, or top number, and the denominator, or bottom number?

Fractions occur often in real-life situations. You see them in slices of pie and pizzas and in sections of citrus fruits.

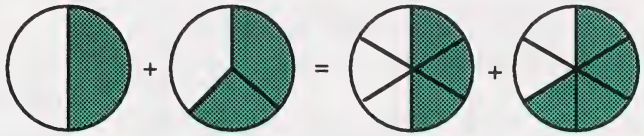
You use fractions when you speak of half an hour or three quarters of an hour. When baking and cooking, you might be asked to add $\frac{1}{2}$ cup of sugar or $\frac{1}{3}$ teaspoon of vanilla in making a cake.

It is important to be able to compute with fractions in order to solve many problems.

Suppose you needed to add $\frac{1}{2}$ hour and $\frac{2}{3}$ hour for some reason. To solve this problem, you need to know how to add fractions.

In order to add or subtract two or more fractions, first make the denominators the same.

Find the lowest common denominator (L.C.D.) to make the denominators the same. For example, the L.C.D. of $\frac{1}{2}$ and $\frac{2}{3}$ is 6. Therefore, change $\frac{1}{2}$ and $\frac{2}{3}$ to equivalent fractions with 6 as the denominator.



$$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6}$$

$$= \frac{7}{6}$$

$$= 1\frac{1}{6}$$

Example 3

- Add $\frac{4}{12}+\frac{7}{12}$.

Solution:

$$\frac{4}{12}+\frac{7}{12}=\frac{4+7}{12} \quad \text{(The L.C.D. is 12.)}$$
$$=\frac{11}{12}$$

This is a basic fraction which cannot be reduced.

- Add $\frac{3}{5}+\frac{4}{15}$.

Solution:

$$\frac{3}{5}+\frac{4}{15}=\frac{3\times3}{5\times3}+\frac{4}{15} \quad \text{(The L.C.D. is 15.)}$$
$$=\frac{9}{15}+\frac{4}{15}$$
$$=\frac{13}{15}$$

- Subtract $\frac{5}{6}-\frac{3}{4}$.

Solution:

$$\frac{5}{6}-\frac{3}{4}=\frac{5\times2}{6\times2}-\frac{3\times3}{4\times3} \quad \text{(The L.C.D. is 12.)}$$
$$=\frac{10}{12}-\frac{9}{12}$$
$$=\frac{1}{12}$$

When the denominators are the same, add or subtract the numerators. Express the sum or difference over the common denominator.

When the denominators are not the same, change one or both fractions so they have a common denominator. Then, add or subtract the numerators. Express the sum or difference over the common denominator.

Example 4

- Solve $1\frac{2}{3}+3\frac{5}{6}$.

Solution:

Method 1

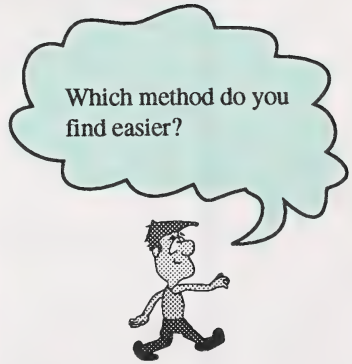
$$1\frac{2}{3}+3\frac{5}{6}=\frac{5}{3}+\frac{23}{6}$$
$$=\frac{5\times2}{3\times2}+\frac{23}{6}$$
$$=\frac{10}{6}+\frac{23}{6}$$
$$=\frac{10+23}{6}$$
$$=\frac{33}{6}$$
$$=5\frac{3}{6}$$
$$=5\frac{1}{2}$$

In Method 1, change the mixed numbers to improper fractions before you add or subtract.

Method 2

$$1\frac{2}{3}=1\frac{2\times2}{3\times2}=1\frac{4}{6}$$
$$+3\frac{5}{6}=3\frac{5\times1}{6\times1}=3\frac{5}{6}$$
$$4\frac{9}{6}=5\frac{3}{6}$$
$$=5\frac{1}{2}$$

In Method 2, the mixed numbers are not changed to improper fractions.



- Solve $31\frac{1}{2}+6\frac{2}{3}$.

Solution:

Method 1

$$31\frac{1}{2}+6\frac{2}{3}=\frac{63}{2}+\frac{20}{3}$$
$$=\frac{63\times3}{2\times3}+\frac{20\times2}{3\times2}$$
$$=\frac{189}{6}+\frac{40}{6}$$
$$=\frac{189+40}{6}=\frac{229}{6}$$
$$=38\frac{1}{6}$$

Method 2

$$31\frac{1}{2} = 31\frac{3}{6}$$
$$+\,6\frac{2}{3} = 6\frac{4}{6}$$
$$37\frac{7}{6} = 37 + 1\frac{1}{6}$$
$$= 38\frac{1}{6}$$

- Solve $31 + 2\frac{3}{4}$.

Solution:

Method 1

$$31 + 2\frac{3}{4} = \frac{31}{1} + \frac{11}{4}$$
$$= \frac{31 \cdot 4}{1 \cdot 4} + \frac{11}{4}$$
$$= \frac{124}{4} + \frac{11}{4}$$
$$= \frac{135}{4}$$
$$= 33\frac{3}{4}$$

Method 2

$$31$$
$$+\,2\frac{3}{4}$$
$$33\frac{3}{4}$$

- Solve $6\frac{1}{2} - 2\frac{1}{3}$.

Solution:

Method 1

$$6\frac{1}{2} - 2\frac{1}{3} = \frac{13}{2} - \frac{7}{3}$$
$$= \frac{13 \cdot 3}{2 \cdot 3} - \frac{7 \cdot 2}{3 \cdot 2}$$
$$= \frac{39}{6} - \frac{14}{6}$$
$$= \frac{25}{6}$$
$$= 4\frac{1}{6}$$

Method 2

$$6\frac{1}{2} = 6\frac{3}{6}$$
$$\underline{-2\frac{1}{3} = 2\frac{2}{6}}$$
$$4\frac{1}{6}$$

- Solve $6\frac{1}{2} - 4$.

Solution:

Method 1

$$6\frac{1}{2} - 4 = \frac{13}{2} - \frac{4}{1}$$
$$= \frac{13}{2} - \frac{4\times 2}{1\times 2}$$
$$= \frac{13}{2} - \frac{8}{2}$$
$$= \frac{5}{2}$$
$$= 2\frac{1}{2}$$

Method 2

$$6\frac{1}{2}$$
$$\underline{-4}$$
$$2\frac{1}{2}$$

- Solve $7 - 3\frac{3}{5}$.

Solution:

Method 1

$$7 - 3\frac{3}{5} = \frac{7}{1} - \frac{18}{5}$$
$$= \frac{7\times 5}{1\times 5} - \frac{18}{5}$$
$$= \frac{35}{5} - \frac{18}{5}$$
$$= \frac{17}{5}$$
$$= 3\frac{2}{5}$$

Method 2

$$7 = 6\frac{5}{5}$$
$$\underline{-3\frac{3}{5} = 3\frac{3}{5}}$$
$$3\frac{2}{5}$$

- Solve $6\frac{2}{3}-4\frac{8}{9}$.

Solution:

Method 1

$$6\frac{2}{3}-4\frac{8}{9}=\frac{20}{3}-\frac{44}{9}$$
$$=\frac{20\times3}{3\times3}-\frac{44}{9}$$
$$=\frac{60}{9}-\frac{44}{9}$$
$$=\frac{16}{9}$$
$$=1\frac{7}{9}$$

Method 2

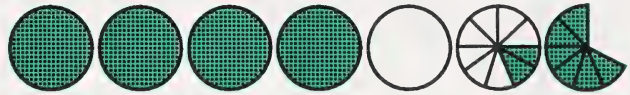
$$6\frac{2}{3}=\boxed{6}\frac{\mathbf{6}}{\mathbf{9}}=\boxed{5}\frac{\mathbf{15}}{9}$$
$$-4\frac{8}{9}=\quad 4\frac{8}{9}=\quad 4\frac{8}{9}$$
$$1\frac{7}{9}$$

The previous diagram shows $6\frac{2}{3}$.

The previous diagram shows $6\frac{2}{3}$ or $6\frac{6}{9}$.

The previous diagram shows $5+1+\frac{6}{9}=5+\frac{9}{9}+\frac{6}{9}$
$=5\frac{15}{9}$.

If four whole circles and eight-ninths of a circle are shaded in, it shows that $4\frac{8}{9}$ is being subtracted from $6\frac{2}{3}$. You will be left with $1\frac{7}{9}$.



Do at least two parts from each of questions 1 to 4.

1. a. $\frac{1}{2}-\frac{5}{12}$ b. $\frac{9}{16}+\frac{3}{4}$

   c. $\frac{1}{2}+\frac{7}{10}$

2. a. $\frac{2}{3}+\frac{3}{4}$ b. $\frac{2}{5}-\frac{2}{15}$

   c. $\frac{3}{4}-\frac{5}{8}$

3. a. $3\frac{1}{2}+1\frac{1}{3}$ b. $5\frac{1}{2}+\frac{7}{8}$

   c. $16+8\frac{1}{2}$

4. a. $3\frac{1}{2}-2$ b. $3\frac{2}{3}-2\frac{5}{6}$

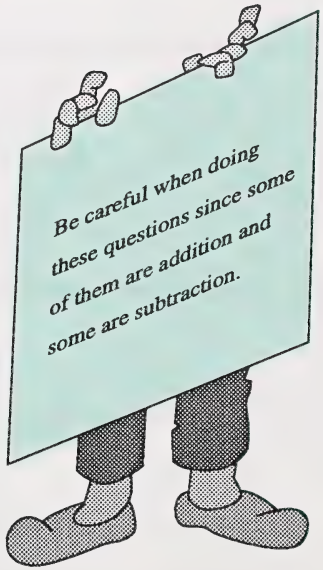
   c. $6-3\frac{7}{8}$

5. Give the L.C.D. for each of the following groups of fractions.

   a. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$ b. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

   c. $\frac{8}{13}, \frac{9}{11}$ d. $\frac{1}{5}, \frac{5}{6}, \frac{7}{8}$

For solutions to **Activity 2**, turn to the **Appendix, Topic 1**.



Activity 3

Perform multiplication and division involving fractions.

When you added and subtracted fractions in the last section, you needed to first find the common denominator. When you multiply fractions, multiply the terms of the numerator; then multiply the terms of the denominator. Then, reduce or simplify the fraction to its lowest form or basic form.

Example 5

- Multiply the fractions $\frac{2}{3}$ and $\frac{1}{2}$.

Solution:

$$\frac{2}{3}\times\frac{1}{2}=\frac{2\times1}{3\times2}$$
$$=\frac{2}{6}$$
$$=\frac{1}{3}$$

- Multiply the fractions $\frac{3}{4}$ and $\frac{8}{3}$.

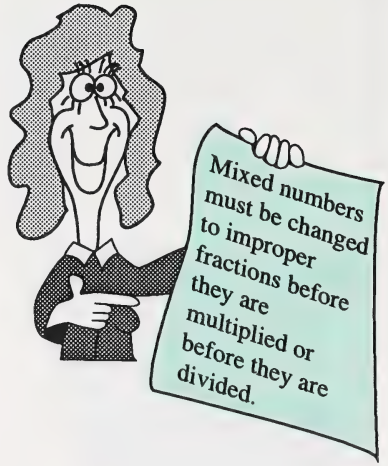
Solution:

$$\frac{3}{4}\times\frac{8}{3}=\frac{3\times8}{4\times3}$$
$$=\frac{24}{12}$$
$$=2$$

- Multiply the mixed numbers $1\frac{1}{2}$ and $2\frac{2}{5}$.

Solution:

$$1\frac{1}{2}\times2\frac{2}{5}=\frac{3}{2}\times\frac{12}{5}$$
$$=\frac{3\times12}{2\times5}$$
$$=\frac{36}{10}$$
$$=3\frac{6}{10}$$
$$=3\frac{3}{5}$$



When dividing fractions, invert the term that follows the division sign; then proceed with multiplication.

When computing $18 \div 6$, simply do the following.

$$\begin{aligned} 18 \div 6 &= 18 \div \frac{6}{1} \\ &= \frac{18}{1} \times \frac{1}{6} \\ &= \frac{18}{6} \\ &= 3 \end{aligned}$$

Invert the number after the division sign; then multiply, and reduce the answer.

Do the same when dividing two mixed numbers.

Example 6

- Solve $\frac{3}{4} \div \frac{2}{3}$.

  Solution:

$$\begin{aligned} \frac{3}{4} \div \frac{2}{3} &= \frac{3}{4} \times \frac{3}{2} \\ &= \frac{3 \times 3}{4 \times 2} \\ &= \frac{9}{8} \\ &= 1\frac{1}{8} \end{aligned}$$

- Solve $1\frac{1}{2} \div \frac{1}{4}$.

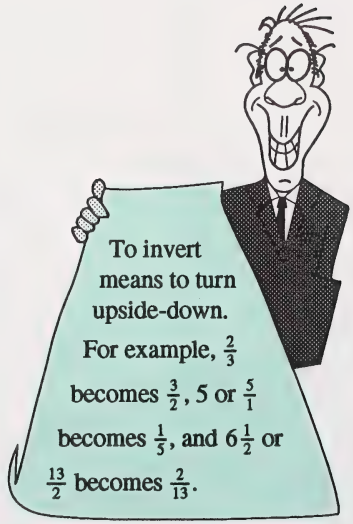
  Solution:

$$\begin{aligned} 1\frac{1}{2} \div \frac{1}{4} &= \frac{3}{2} \div \frac{1}{4} \\ &= \frac{3}{2} \times \frac{4}{1} \\ &= \frac{12}{2} \\ &= 6 \end{aligned}$$

- Solve $2\frac{2}{3} \div \frac{2}{9}$.

  Solution:

$$\begin{aligned} 2\frac{2}{3} \div \frac{2}{9} &= \frac{8}{3} \div \frac{2}{9} \\ &= \frac{8}{3} \times \frac{9}{2} \\ &= \frac{72}{6} \\ &= 12 \end{aligned}$$


To invert means to turn upside-down.
For example, $\frac{2}{3}$ becomes $\frac{3}{2}$, 5 or $\frac{5}{1}$ becomes $\frac{1}{5}$, and $6\frac{1}{2}$ or $\frac{13}{2}$ becomes $\frac{2}{13}$.

Do any three of the following exercises.

1. a. $\frac{2}{5}\times\frac{3}{4}$ b. $\frac{3}{7}\times\frac{3}{10}$

   c. $1\frac{5}{9}\times 2\frac{1}{3}$

2. a. $2\frac{7}{10}\times 6\frac{2}{3}$ b. $\frac{6}{5}\times 3\frac{1}{2}$

   c. $7+2\frac{1}{3}$

3. a. $5+1\frac{1}{2}$ b. $6\frac{1}{2}+3$

   c. $18\div 27$

4. Similar books are piled to a height of 45 cm. If one book is $2\frac{1}{2}$ cm thick, how many books are in the pile?

5. A gasoline tank has a capacity of 72 litres. The tank is $\frac{3}{4}$ full. How much gasoline is in the tank?

For solutions to **Activity 3**, turn to the **Appendix**, **Topic 1**.

Activity 4

Perform addition and subtraction involving decimals.

In earlier grades you learned about decimal numbers. In this section you shall learn and use a few more skills needed when adding and subtracting decimal numbers.

When adding or subtracting natural numbers, write the respective digits in columns. Write all the ones digits in the same column and do the same for the tens digits, the hundreds digits, and so on. When adding or subtracting decimal numbers, do the same. However, you have to be a bit more careful since you have a decimal point to keep track of.

The rule is that you always line up the decimal points.

Example 7

- Find the sum of $0.56+32.4+0.315$.

Solution:

Align the digits according to their place value. A way to ensure proper alignment is shown at the left.

	0	.	5	6	
3	2	.	4		
	0	.	3	1	5
3	3	.	2	7	5

0.560
32.400
0.315
33.275

- Find the sum of
  $31.15 + $13.00 + $0.35 + $15.06.

  Solution:

  The dollar is a decimal currency; therefore, add the dollars in the same way as other decimal numbers.

$$\begin{array}{r} \$\ 31.15 \\ \$\ 13.00 \\ \$\ \ 0.35 \\ +\ \$\ 15.06 \\ \hline \$\ 59.56 \end{array}$$

When subtracting decimal numbers, proceed in the same way as adding decimal numbers. Align the decimals and subtract ones from ones, tens from tens, and so on.

Look at a few examples of subtraction of decimal numbers.

Example 8

- Solve 301.024 – 243.86.

  Solution:

$$\begin{array}{r} 301.024 \\ -243.860 \\ \hline 57.164 \end{array}$$

- Solve 3.045 – 0.96.

  Solution:

$$\begin{array}{r} 3.045 \\ -0.960 \\ \hline 2.085 \end{array}$$

- Solve 0.45 – 0.006.

  Solution:

$$\begin{array}{r} 0.450 \\ -0.006 \\ \hline 0.444 \end{array}$$

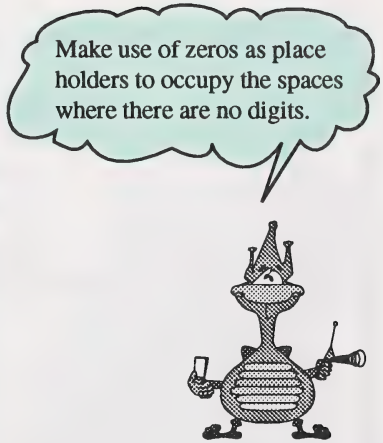
Do any four of the following questions.

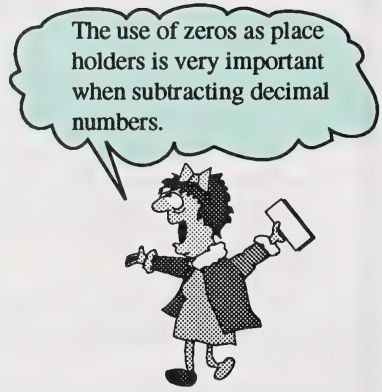
1. a. 23.014 + 8.39 + 1.4

   b. 0.53 + 0.9 + 0.008

   c. 164.43 + 0.697 + 8.5654

2. Workers on an assembly line are paid by the hour. How much would five workers earn altogether in an hour if they are paid the following hourly wages: $3.85, $5.05, $4.95, $3.50, and $5.95?





3. Do the following calculations.

   a. 3.157 – 2.5

   b. 1.0250 – 0.98

   c. 165 – 95.625

4. Evaluate the following:

   a. 62.455 – 29.09

   b. 1.9 – 0.002

   c. 170.05 – 78.651

5. Find the perimeter of a farmer's field if the sides are 21.9 m, 25.05 m, 101.5 m, 9.25 m, and 65 m.

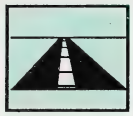
6. a. Subtract 11.9 from 18.006.

   b. Subtract 0.0129 from 0.9034.

   c. Subtract 245.208 from 343.08.

For solutions to **Activity 4**, turn to the **Appendix, Topic 1**.

Activity 5



Perform multiplication and division of decimals.

Multiplication of decimals is similar to multiplication of natural numbers. The main difference is being able to place the decimal point in the right place.

In order to determine the number of decimal places, count the number of digits to the right of the decimal point.

For example, you should know that there are four decimal places in 4.4213, there are two decimal places in 0.03, and there are three decimal places in 343.469. How many decimal places are there in the following?

- 3.2

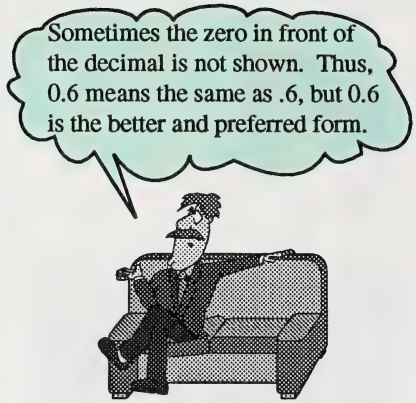
  There is one decimal place.

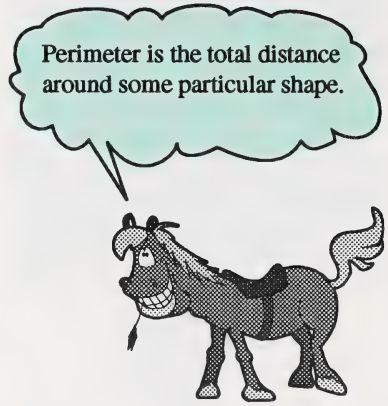
- 148

  There are no decimal places.

- 140.006

  There are three decimal places.





- 4.1

  There is one decimal place.

- 0.01

  There are two decimal places.

- 23.4001

  There are four decimal places.

Remember, when you multiply two decimal numbers, your answer must have the same number of decimal places as the total of the decimal places in the two decimal numbers you multiplied.

For example, the answer for 2.3×4.8 will have two decimal places, and the answer for 71.03×0.421 will have five decimal places.

Example 9

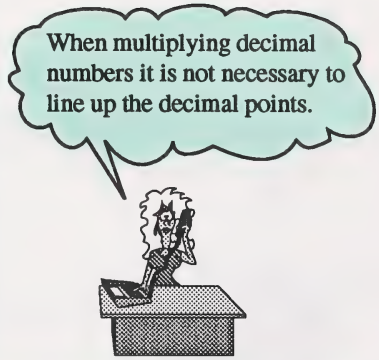
- Multiply 22.02 by 0.03.

  Solution:

$$\begin{array}{r} 22.02 \\ \times\ 0.03 \\ \hline 0.6606 \end{array}$$

(1) (2) (3) (4) above the factors; (1) (2) (3) (4) below the product

There are four decimal places in total in the original numbers. Therefore, there are four decimal places in the answer.



Follow this rule. Multiply the numbers as if there were no decimal points. Then, place the decimal point in the answer so that the number of digits to the right of the decimal point in the product is equal to the sum of the number of digits to the right of the decimal point in each factor. Factors are the numbers that are being multiplied. The product is the answer to any multiplication problem.

- Multiply 48.01 by 3.9.

Solution:

$$\begin{array}{r} 48.01 \\ \times 3.9 \\ \hline 43209 \\ 14403 \\ \hline 187.239 \end{array}$$

- Multiply 0.12 by 0.305.

Solution:

$$\begin{array}{r} 0.305 \\ \times 0.12 \\ \hline 610 \\ 305 \\ \hline 0.03660 \end{array}$$

If you have problems with the decimal point, you could disregard it until the end. For example, if you multiply 0.004 by 0.012, first multiply $4 \times 12 = 48$. Then, count the decimal places in the original numbers. (There are six.) Your answer must have six digits after the decimal point. The answer will be 0.000 048. Simply insert the decimal in the right place. Notice that enough zeros are used to make the movement of the decimal point possible.

Division using decimal numbers occurs frequently in everyday life so you should know how to deal with this operation.

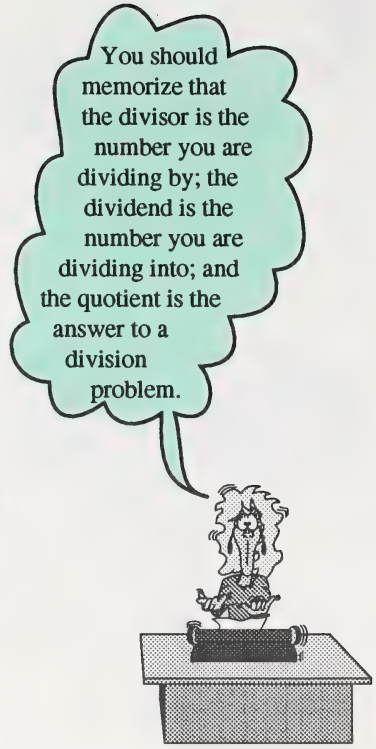
First, you could say a fraction indicates division.

If you write $\frac{7}{8}$ you are saying that 7 is divided by 8 or $7 \div 8$. The result of dividing through completely will be a decimal number.

$$\begin{array}{r} 0.875 \\ 8\overline{)\,7.000} \\ 64 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

When dividing one decimal number by another, there are certain steps to follow to avoid confusion and unnecessary difficulties.

For example, divide 4.9 by 0.14. First, remove the decimal point from the divisor. In other words change the divisor to a whole number.


You should memorize that the divisor is the number you are dividing by; the dividend is the number you are dividing into; and the quotient is the answer to a division problem.

Write the expression as a fraction which gives $\frac{4.9}{0.14}$. Then move the decimal point two places to the right in both parts. In effect you are really multiplying the numerator by 100 and the denominator by 100.

$$\frac{4.9}{0.14} \times \frac{100}{100} = \frac{490.0}{14.0}$$

Now divide.

$$\begin{array}{r} 35.0 \\ 14\overline{)490.0} \\ \underline{42} \\ 70 \\ \underline{70} \\ 0 \end{array}$$

Hence, $4.9 \div 0.14 = 35.0$ or 35.

Example 10

Do the following correct to three decimal places.

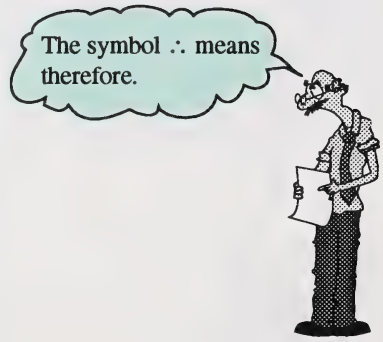
- Calculate $10.24 \div 0.8$.

Solution:

$10.24 \div 0.8$ becomes $\frac{10.24}{0.8}$ or $\frac{102.4}{8}$.

$$\begin{array}{r} 12.8 \\ 8\overline{)102.4} \\ \underline{8} \\ 22 \\ \underline{16} \\ 64 \\ \underline{64} \\ 0 \end{array}$$

$\therefore 10.24 \div 0.8 = 12.8$



- Calculate $27.81 \div 4.5$.

Solution:

$27.81 \div 4.5 = 278.1 \div 45$

Move the decimal one place to the right in the divisor and the dividend.

$$
\begin{array}{r}
6.18 \\
45\overline{)\,278.10} \\
270.0 \\
\hline
81 \\
45 \\
\hline
360 \\
360 \\
\hline
0
\end{array}
$$

$\therefore\ 27.81 \div 4.5 = 6.18$

Note that you can add zeros after the decimal point if you need to do so.

- Calculate $0.5719 \div 3.8$.

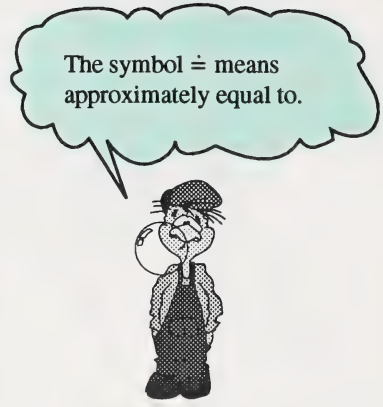
Solution:

Move the decimal one place to the right.

$$
\begin{array}{r}
0.1505 \\
38\overline{)\,5.7190} \\
38 \\
\hline
191 \\
190 \\
\hline
19 \\
0 \\
\hline
190 \\
190 \\
\hline
0
\end{array}
$$

$0.5719 \div 3.8 = 0.1505$

Therefore, $0.5719 \div 3.8 = 0.151$ correct to three decimal places.



- Calculate $0.49 \div 0.343$.

  Solution:

  Move the decimal three places to the right in both numbers.

$$
\begin{array}{r}
1.4285 \\
343\overline{)490.0000} \\
343.0 \\
\hline
1470 \\
1372 \\
\hline
980 \\
686 \\
\hline
2940 \\
2744 \\
\hline
1960 \\
1716 \\
\hline
244
\end{array}
$$

  Therefore, $0.49 \div 0.343 \doteq 1.429$ correct to three decimal places.

Now it is time for you to practice these operations.

Do any five of the following.

1. Put the decimal in the right place for the following products.

   a. $0.025 \times 0.135 = 3375$

   b. $2.75 \times 3.2 = 8800$

   c. $5.7 \times 0.63 = 3591$

   d. $3.3 \times 4.5 = 1485$

   e. $1.25 \times 33.6 = 42\,000$

   f. $0.413 \times 17.5 = 72\,275$

2. Find the products.

   a. 1.35×6.23 b. 5.25×0.75

   c. 0.25×0.125 d. 28.7×16.15

   e. 276.4×0.825

3. Divide the following. Round your answers to three decimal places where applicable.

   a. $1.65 \div 12$

   b. $0.087 \div 2.8$

   c. $52.13 \div 2.08$

4. Find the quotients. Round your answer to four decimal places.

   a. $735.45 \div 25.7$

   b. $49.0 \div 2.401$

   c. $0.032 \div 10.24$

5. Evaluate each of the following. Round your answers to three decimal places where applicable.

   a. $14.44 \div 1.44$

   b. 5.6×17.1

   c. 48.0×0.66

6. Find the area of a square chess board that has a side of 38.02 cm.

7. The area of a rectangular field is 772.65 m^2. One side is 25.5 m. What is the length of the other side?

For solutions to **Activity 5**, turn to the **Appendix**, **Topic 1**.

Activity 6

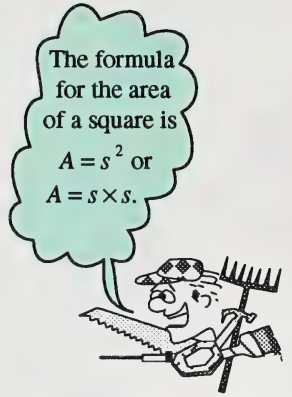
Round and estimate with decimals.

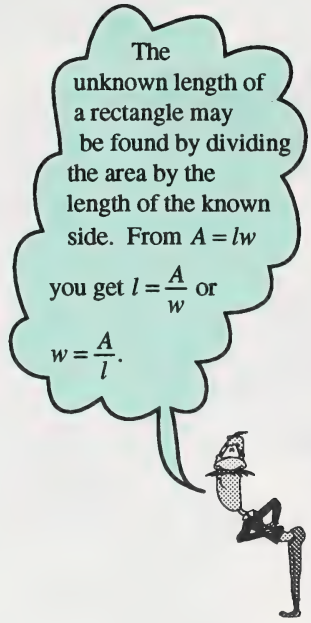
When teachers grade their students, they usually round to the nearest whole number. Some teachers round to the nearest multiple of five. In athletic events, time is often measured to the nearest tenth or hundredth of a second.

Now that calculators are common, you will notice that there is a greater need for rounding numbers. For example, your calculator will tell you that $1 \div 7 = 0.142\,8571\ldots$

You might only need two or three decimal places for your particular purpose.

Rounding is a procedure which will enable you to give an acceptable approximation for a measurement or a degree of accuracy of a number when an exact value is impractical or impossible to determine.




The unknown length of a rectangle may be found by dividing the area by the length of the known side. From $A = lw$ you get $l = \frac{A}{w}$ or $w = \frac{A}{l}$.

Scientists do a lot of rounding and estimating due to the size and magnitude of the objects they have to measure and quantify. Consider the case of the moon orbiting the Earth. When scientists tell us that the moon's orbit is 28 days, they are rounding the actual amount of time to the nearest day. When they say the distance from the Earth to the Sun is 150 million kilometres, they are rounding to the nearest ten million kilometres.

An important reason to know about rounding is for estimation.

An estimate is a reasonable guess as to what the outcome will be.

Can you use a calculator and then tell if your answer is reasonable? Yes, you can.

To make an estimate, first round off the numbers to a specified point. Then calculate using these rounded numbers. The estimate should be approximately equal to the exact answer. Often, calculations with rounded numbers are done mentally. These examples will show you how.

The number of years required for Neptune to revolve around the sun is 164.789. This decimal can be rounded to the nearest

- hundredth
- tenth
- ones or units

The number 164.789 rounded to

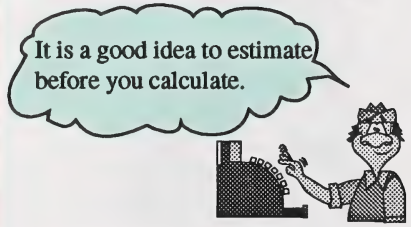
- the nearest hundredth is 164.79
- the nearest tenth is 164.8
- the nearest one is 165

If you are rounding to the nearest hundredth, look at the thousandths digit. If it is 4, 3, 2, 1, or 0, discard this digit and all other digits to the right. If the thousandths digit is 5, 6, 7, 8, or 9, increase the hundredths digit by 1 and drop the rest of the digits which follow it. The number 63.073 is rounded to 63.07 and the number 54.399 is rounded to 54.40 if you are rounding to the nearest hundredth.

Here are more examples to demonstrate this. It is not as difficult as it appears at first.

Each of the following pair of statements means the same thing.

- Round off to the nearest hundredth.
  or
  Round off to two decimal places.
- Round off to the nearest thousandth.
  or
  Round off to three decimal places.



When 54.399 is rounded to the nearest hundredth, the 0 must be kept at the end to indicate hundredths. To the nearest hundredth, 54.399 is 54.40 not 54.4.

Example 11

Round the number 642.9425 to the following.

- thousandth

  Solution:

  Since the ten thousandths place is equal to 5, 642.9425 rounded to the nearest thousandth is equal to 642.943.

- hundredth

  Solution:

  Since the thousandths place is less than 5, 642.9425 rounded to the nearest hundredth is equal to 642.94.

- tenth

  Solution:

  Since the hundredths place is less than 5, 642.9425 rounded to the nearst tenth is equal to 642.9.

- one (unit)

  Solution:

  Since the tenths place is greater than 5, 642.9425 rounded to the nearest one is equal to 643.

- ten

  Solution:

  Since the units place is less than 5, 642.9425 rounded to the nearest ten is equal to 640.

- hundred

  Solution:

  Since the tens place is less than 5, 642.9425 rounded to the nearest hundred is equal to 600.

Do you remember place value? The following is a quick review.

6	4	2	.	9	4	2	5
↓	↓	↓		↓	↓	↓	↓
hundreds	tens	units or ones		tenths	hundredths	thousandths	ten thousandths

If the number to the right of the place to which you are rounding is 5 or over, round up. If it is less than 5, leave it as it is presently.

Observe this pattern when rounding the following numbers to the tenths place.

72.80, 72.81, 72.82 } becomes 72.8
72.83, 72.84

72.85, 72.86, 72.87 } becomes 72.9
72.88, 72.89

Note that digits after the hundredths place won't make a difference when rounding to the nearest tenth.

The use of rounded numbers is very convenient in estimating a reasonable answer in problem solving. For some problems, an estimate gives you an answer that is not precise but it is adequate.

People like tailors, caterers, dressmakers, and merchants use estimation. Do you ever estimate?

Example 12

Floyd Ineisbach would like to gravel his driveway to a depth of 15 cm. If his driveway is 50 m long and 5 m wide, estimate how many loads of gravel would be needed to cover his driveway? (A load of gravel is 10 m^3.)

Solution:

The volume of gravel Floyd would need for the driveway is found by using the volume formula which is $V = l \times w \times h$. (The 15 cm depth equals 0.15 m.)

$$V = 50 \text{ cm} \times 5 \text{ m} \times 0.15 \text{ m}$$
$$= 37.50 \text{ m}^3$$

A load of gravel = 10 m^3. Therefore, an estimate of the number of loads of gravel to cover the driveway would be $37.50 \div 10 = 3.75$ or approximately four loads.

Floyd will need about four loads of gravel for his driveway.

The symbol $\doteq$ means approximately equal to.

The following is an example of rounding to the nearest whole number.

$37.50 \div 10 = 3.750$
$\doteq 4$

Example 13

Estimate 306×492.

Solution:

Round 306 to 300 and 492 to 500. Since $300 \times 500 = 150\,000$ we know that 306×492 must be close to 150 000.

Do any five of the following questions.

1. What is the difference between rounding to the nearest thousand and rounding to the nearest thousandth?

2. Round each of the following to the nearest tenth and to the nearest one (or units).

   a. 491.99 b. 222.95

   c. 95.59 d. 45.08

   e. 19.473 f. 9.849

3. Round each of the following to the nearest dollar and to the nearest cent.

   a. \$219.09 b. \$0.956

   c. \$250.49 d. \$111.974

   e. \$343.14

4. Give a reasonable estimate for the following products.

   a. 301×497 b. 949×101

   c. 0.91×0.502 d. 298.4×209.87

   e. 3.05×15.004

5. Joe Wright works nine hours a day at an answering service. If he is paid \$9.58 per hour, estimate how much he earns per week if he works five days a week.

6. Miss Polet reads 21 pages of a book in 49 minutes. Estimate how long it will take her to read the 430 pages of the book?

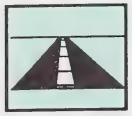
7. Estimate the solution for each of the following.

   a. $\dfrac{9.8 \times 19.7 \times 51.01}{2.1 \times 4.9}$ b. $\dfrac{49 \times 501}{2.5}$

   c. $39\dfrac{30}{31} \times 30\dfrac{1}{29}$

For solutions to **Activity 6**, turn to the **Appendix**, **Topic 1**.

Activity 7



Change fractions to decimals.

Very often it is necessary to convert a fraction to a decimal number. With a calculator you can quickly perform this conversion.

Look at the following rule.

> To find the decimal number equivalent of a fraction, divide the numerator by the denominator.

Example 14

Find the decimal number equivalent of $\frac{3}{8}$.

Solution:

By definition, $\frac{3}{8} = 3 \div 8$.

Now divide.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Therefore, $\frac{3}{8}$ is equivalent to 0.375.

Example 15

- About $\frac{3}{4}$ of all motor vehicles registered in Canada are cars. The rational number $\frac{3}{4}$ can be expressed as a decimal by dividing 3 by 4.

Solution:

Enter	Display
3	3
÷	3
4	4
=	0.75

or

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

As a decimal number, $\frac{3}{4} = 0.75$. The division ends because the remainder is 0. A decimal number such as 0.75 is called a **terminating decimal**.

- Trucks and buses together make up about $\frac{3}{11}$ of all vehicles registered in Canada.

Solution:

Enter	Display
3	3
+	3
11	11
=	0.2727...

$$
\begin{array}{r}
0.2727 \\
11\overline{)3.0000} \\
22 \\
\hline
80 \\
77 \\
\hline
30 \\
22 \\
\hline
80 \\
77 \\
\hline
3
\end{array}
$$

The division does not end. The quotient has digits that repeat. The decimal number such as 0.2727... is called a **repeating decimal**. Repeating decimal numbers are often written using bar notation or dot notation.

An example of bar notation is $0.2727\ldots = 0.\overline{27}$. An example of dot notation is $0.2727\ldots = 0.\dot{2}\dot{7}$. In $0.\overline{27}$, 27 repeats. 27 is called the **period**. Since two digits repeat, the **length of the period is 2.**

Example 16

Express $4\frac{2}{3}$ as a decimal number.

Solution:

Since $4\frac{2}{3} = \frac{14}{3}$, you can divide as shown here.

$$
\begin{array}{r}
4.666 \\
3\overline{)14.000} \\
12 \\
\hline
20 \\
18 \\
\hline
20 \\
18 \\
\hline
20 \\
18 \\
\hline
2
\end{array}
$$

The decimal number equivalent of $4\frac{2}{3}$ is $4.666\ldots = 4.\overline{6}$. (Note that the decimal number equivalent repeats from the tenths place onward.)

A terminating decimal is a decimal that comes to an end. One example would be 4.728.

Repeating decimals are decimals that have one or more repeating digits. Several examples would be 2.424 242... or $2.\overline{42}$ or $2.\dot{4}\dot{2}$ and 6.33... or $6.\overline{3}$ or $6.\dot{3}$.

Example 17

Write a decimal number for $\frac{2}{7}$. Round to the nearest ten thousandth.

Solution:

$$
\begin{array}{r}
0.28571 \\
7\overline{)2.00000} \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
10 \\
7 \\
\hline
3
\end{array}
$$

The decimal number for $\frac{2}{7}$ rounded to the nearest ten thousandth is 0.2857.

Remember these key points:

- $4 \div 3$ is the same as $4.000 \div 3$.
- A repeating decimal number like 1.4444... can be written as $1.\overline{4}$ or $1.\dot{4}$, and 2.4545... as $2.\overline{45}$ or $2.\dot{4}\dot{5}$.
- If the numerator is greater than the denominator you have an improper fraction as in $\frac{22}{7}$.
- A fraction with a numerator of zero is itself equal to zero as in $\frac{0}{5} = 0$.
- You cannot divide a number by zero. For example, $\frac{6}{0}$ is not defined. Try it on a calculator!

Try some of the following exercises.

Do four of the following:

1. Change the following fractions to decimals.

   a. $\frac{1}{2}$ b. $\frac{1}{4}$

   c. $\frac{3}{5}$ d. $\frac{3}{4}$

   e. $\frac{4}{5}$

2. Write each as a decimal.

   a. $\frac{1}{8}$ b. $2\frac{1}{2}$

   c. $7\frac{1}{4}$ d. $1\frac{7}{8}$

   e. $2\frac{2}{5}$

3. Give the decimal equivalent of the following. Round each answer to three decimal places.

   a. $4\frac{1}{3}$ b. $3\frac{1}{7}$

   c. $1\frac{5}{11}$ d. $\frac{8}{9}$

   e. $2\frac{5}{6}$

4. Change each of the following fractions to decimals.

   a. $\frac{0}{5}$ b. $\frac{83}{1000}$

   c. $1\frac{7}{20}$ d. $\frac{23}{40}$

   e. $\frac{273}{100}$

5. Express the following as a decimal where possible.

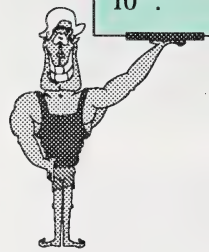
   a. 17 b. $\frac{5}{0}$

   c. 2^3 d. $\frac{2.5}{4}$

   e. $\frac{0}{0}$

For solutions to **Activity 7**, turn to the **Appendix, Topic 1**.

When dividing by powers of 10, move the decimal point in the numerator to the left as many times as there are zeros in the power itself. Several examples are $25 \div 100 = 0.25$ and $3 \div 10\,000 = 0.0003$. As a power, 100 is 10×10 or 10^2. As a power, 10 000 is $10 \times 10 \times 10 \times 10$ or 10^4.



Activity 8

Change decimals to fractions.

Now that you know how to write fractions as decimals, look at how to do the reverse process.

Example 18

- Write 0.6 as a fraction.

  Solution:

$$0.6 = \frac{6}{10} \quad \text{(G.C.F. for 6 and 10 is 2.)}$$
$$= \frac{3}{5}$$

- Write 0.25 as a fraction.

  Solution:

$$0.25 = \frac{25}{100} \quad \text{(G.C.F. for 25 and 100 is 25.)}$$
$$= \frac{1}{4}$$

- Write 0.375 as a fraction.

  Solution:

$$0.375 = \frac{375}{1000} \quad \text{(G.C.F. for 375 and 1000 is 125.)}$$
$$= \frac{3}{8}$$

You will notice the following from the previous example.

- If the decimal number is in tenths (one decimal place), the denominator of the fraction is 10.
- If the decimal number is in hundredths (two decimal places), the denominator of the fraction is 100.
- If the decimal is in thousandths (three decimal places), the denominator of the fraction is 1000.

This is a quick review of place value.

6	4	2	.	9	4	2	5	7	8
hundreds (6×100)	tens (4×10)	ones (2×1)		tenths ($9 \times \frac{1}{10}$)	hundredths ($4 \times \frac{1}{100}$)	thousandths ($2 \times \frac{1}{1\,000}$)	ten thousandths ($5 \times \frac{1}{10\,000}$)	hundred thousandths ($7 \times \frac{1}{100\,000}$)	millionths ($8 \times \frac{1}{1\,000\,000}$)

Write one under the decimal and a zero for each digit after the decimal point. Find the G.C.F. and reduce the fraction.

$$0.06 = \frac{6}{100}$$
$$= \frac{3}{50}$$

G.C.F. for 6 and 100 is 2. This pattern is used throughout.

You may write 0.256 as $(0)+\left(2\times\frac{1}{10}\right)+\left(5\times\frac{1}{100}\right)+\left(6\times\frac{1}{1000}\right)$ or as $0+\frac{2}{10}+\frac{5}{100}+\frac{6}{1000}$. When you add these together, you get $0+\frac{200}{1000}+\frac{50}{1000}+\frac{6}{1000}=\frac{256}{1000}$.

Since the L.C.D. = 1000, you can reduce by eliminating the common factor which is 8 and 0.256 becomes $\frac{256\div 8}{1000\div 8}=\frac{32}{125}$.

The decimal number 0.56 written in expanded notation would be $0+\frac{5}{10}+\frac{6}{100}$.

Example 19

- Write 3.268 in expanded notation.

  Solution:

$$3.268=3+\frac{2}{10}+\frac{6}{100}+\frac{8}{1000}$$

- Write 0.0435 in expanded notation.

  Solution:

$$0.0435=0+\frac{0}{10}+\frac{4}{100}+\frac{3}{1000}+\frac{5}{10\,000}$$

- Write 387.4049 in expanded notation.

  Solution:

$$387.4049=300+80+7+\frac{4}{10}+\frac{0}{100}+\frac{4}{1000}+\frac{9}{10\,000}$$

Try a few of the following exercises of this type.

The abbreviation L.C.D. stands for lowest common denominator.

Do any ten from question 1 and any three from each of questions 2 and 3.

1. Write the following decimals as basic fractions.

   a. 0.6 b. 0.25 c. 0.75

   d. 0.1 e. 1.5 f. 3.2

   g. 2.25 h. 7.3 i. 0.125

   j. 1.375 k. 2.45 l. 3.005

2. Write the following in expanded notation.

   a. 3.125 b. 0.3205

   c. 434.525 d. 47.0069

3. Write each as a fraction or a mixed number.

   a. 0.02 b. 2.035

   c. 22.64 d. 0.0005

   e. 1.1234

For solutions to **Activity 8** turn to the **Appendix, Topic 1**.

Activity 9

Perform calculations using the order of operations.

If you have a calculation problem that involves more than one type of operation, you will need a rule that states the order in which these operations are carried out.

You need to study the following **Rules for the Order of Operations**.

1. Do calculations in brackets first.
2. Do all calculations involving exponents next.
3. Multiply and divide in the order that these operations appear in from left to right.
4. Add and subtract in the order that these operations appear in from left to right.

Try to calculate $8 - 6 \div 2$.

What did you get for an answer?

Did you subtract first to get $8 - 6 + 2 = 2 + 2$

$= 1$?

Basic fractions are fractions that can not be simplified or reduced any further. In a basic fraction, the G.C.F. for the numerator and the denominator is 1.

The fraction $\frac{9}{10}$ is a basic fraction because the G.C.F. for 9 and 10 is 1.

When getting dressed, does one put a tie on first or does one start by putting on a shirt first? Does the order of doing this matter?

Did you divide first to get $8-6+2=8-3$

$=5?$

The second way is the correct way to do this exercise. You have to perform the division before doing the subtraction.

An expression may contain one or both of the following.

- operations with brackets
- operations with exponents

The following rule for ordering is used.

Carry out the operation within the brackets before evaluating the exponent. In order to put these rules together you may use, as a memory device, the letters BEDMAS, which states the order of operations as follows.

B - Brackets
E - Exponents
D - Division and
M - Multiplication in the order that they appear
A - Addition and
S - Subtraction in the order that they appear

The following examples demonstrate the BEDMAS rule for the order of operations. Never do more than one operation at a time.

Example 20

Simplify $17+5\times9+3$.

Solution:

$$17+5\times9+3=17+45+3$$
$$=17+15$$
$$=32$$

Example 21

Simplify $2(5\times3^2)-7$.

Solution:

$$2(5\times3^2)-7=2(5\times9)-7$$
$$=2(45)-7$$
$$=90-7$$
$$=83$$

or

$$2(5\times3^2)-7=2\times5\times3^2-7$$
$$=2\times5\times9-7$$
$$=10\times9-7$$
$$=90-7$$
$$=83$$

Here you can do the exponent before removing the brackets.

When posting a letter, does one mail the letter first or is the stamp affixed first? Does the order of doing this matter?

Example 22

Simplify $\dfrac{5\times3^2+3+17}{2^6-2\times3^2+14}$.

Solution:

$$\frac{5\times3^2+3+17}{2^6-2\times3^2+14}=\frac{5\times9+3+17}{64-2\times9+14}$$ (Do the exponents.)

$$=\frac{45+3+17}{64-2\times9+14}$$ (Multiply.)

$$=\frac{15+17}{64-18+14}$$ (Divide.)

$$=\frac{32}{60}$$ (Add and subtract.)

$$=\frac{8}{15}$$ (Simplify.)

After this you are ready for some exercises. Do these questions following the BEDMAS rule.

Do the odd or the even parts in each question. Do all of them if you feel you need more practice.

1. a. $\left(3^2+5\right)+7$ b. $\left(5^2-3\right)\times2$

   c. $4^3+6^2+12-3^3$ d. $3\left(5\times3^2\right)-35$

2. a. $16+8+2^3$ b. $6\times5+7^2$

   c. $36+3^2+12$ d. $4\times5^2-6^2$

3. a. $8\times(7-2)^2+2$ b. $3^4+9+(5+4)$

   c. $\left(18-4^2\right)+2$ d. $3^3+9+(5\times4)$

4. a. $\dfrac{5\left(5^2-5\right)}{2^2+1}$ b. $\dfrac{4\times5^2-6^2}{3^2-1}$

   c. $\dfrac{\left(4^2+2\right)^2}{(3+1)+2^3}$ d. $\dfrac{20-8+2^2}{3\times2^2-4}$

5. a. $207\times1.3-1.51$ b. $1.124+4+0.25$

   c. $14.1+7.9\times3.4-13.72$

6. Evaluate the following.

   a. $4\times50+(80+8)$ b. $49-9+3\times(24+6)\,4$

   c. $(7\times2)-(12+2)-(15+3)$ d. $\dfrac{22+11\times6}{4+4+2}$

For solutions to **Activity 9**, turn to the **Appendix**, **Topic 1**.

Activity 10

Perform calculations involving integers.

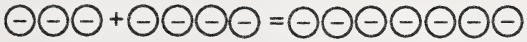
In Activity 1, you reviewed integers. Now you will learn how to perform calculations using integers. Keep in mind that + 3 is the same as 3. In most instances, the + sign is left out when a number is positive.

Adding Integers

- Rule 1

If the signs are the same, add the numbers and attach the common sign.

$-3+(-4)=-7$



$+3+(+4)=+7$

- Rule 2

If the signs are different, subtract the numbers and attach the sign which goes with the number farther from the zero point.

$-3+(+4)=+1$

↓

Farther from the zero point so the sum is +1 not –1.

Three negatives will cancel three positives and you will be left with one positive.

$+3+(-4)=-1$

↓

Farther from the zero point so the sum is –1 not +1.

Three positives will cancel three negatives and you will be left with one negative.

Subtracting Integers

Change the subtraction sign to an addition sign and change the sign of the second number to its opposite. Then follow the rules for addition. Remember that subtraction is the opposite of addition, and addition is the opposite of subtraction.

- $-3-(+3)=-3+(-3)$
  $=-6$

- $+3-(-4)=+3+(+4)$
  $=+7$

- $+4-(+3)=+4+(-3)$
  $=+1$

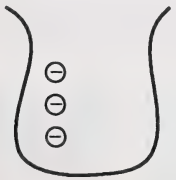
- $-4-(-3)=-4+(+3)$
  $=-1$

To show subtraction using a positive disc ⊕ and a negative disc ⊖, you must know how to apply and use the zero property. When adding one ⊕ and one ⊖ you are really adding zero since one ⊕ combined with one ⊖ is nothing. Zero added to any number does not change that number.

Study the following diagrams which use ⊕ and ⊖ to illustrate the subtraction of integers. Different colored marbles could also be used. For example, let a black marble represent $+1$ and a white marble represent -1. Two colors of paper strips would also work.

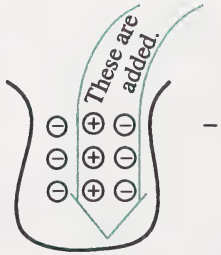
- $-3-(+3)=$

Start with three ⊖.



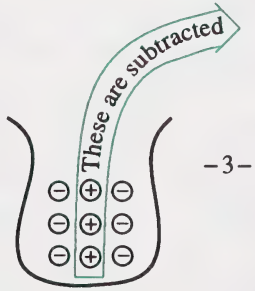
-3

Add three ⊕ and three ⊖. You are really adding zero because of the zero property.

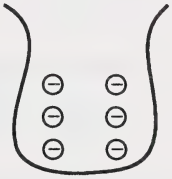


$-3+[+3+(-3)]=-3+0$
$=-3$

Remove or subtract the three ⊕.



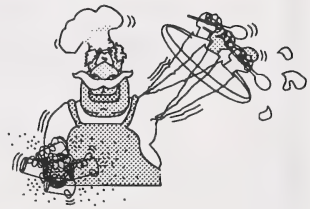
$-3-(+3)$



-6

There are six ⊖ left.
Therefore, $-3-(+3)=-6$.

Avoid a mix-up and confusion. When doing the operation of subtraction, never change the sign of the first number in the setup of the problem.

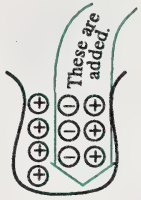


• $+4-(+3)=$

Start with four ⊕.

$+4$

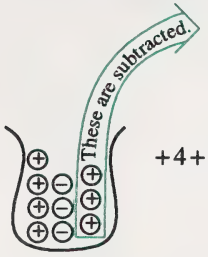
Add three ⊕ and three ⊖. You are really adding zero because of the zero property.



$+4+[-3+(+3)]=+4+0$

$=+4$

Remove or subtract three ⊕.



$+4+[-3+(+3)]$

Three ⊕ will cancel three ⊖ when put together.

$+4+(-3)$

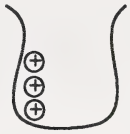
There is one ⊕ left.

$+1$

Therefore, $+4-(+3)=+1$.

• $+3-(-4)=$

Start with three ⊕.



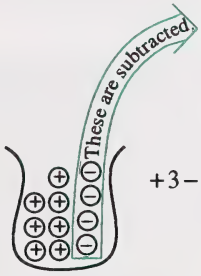
$+3$

Add four ⊕ and four ⊖. You are really adding zero because of the zero property.

$+3+[+4+(-4)]=+3+0$

$=+3$

Remove or subtract four ⊖.



$+3-(-4)$

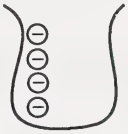
There are seven ⊕ left.

$+7$

Therefore, $+3-(-4)=+7$.

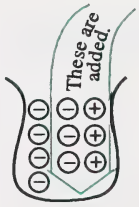
- $-4-(-3)=$

Start with four ⊖.



-4

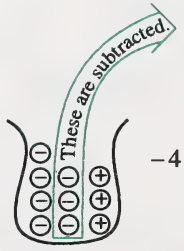
Add three ⊖ and three ⊕. You are really adding zero because of the zero property.



$-4+[-3+(+3)]=-4+0$

$=-4$

Remove or subtract three ⊖.



$-4-(-3)$

Three ⊖ will cancel three ⊕ when put together.

$-4+(+3)$

There is one ⊖ left.

-1

Therefore, $-4-(-3)=-1$.

Multiplying and Dividing Integers

The rules for multiplying integers and dividing integers are the same.

Rule 1

If the signs are the same, the answer is positive.

- $+3\times(+2)=+6$
- $-3\times(-2)=+6$
- $-6\div(-2)=+3$
- $+6\div(+2)=+3$

Rule 2

If the signs are different, the answer is negative.

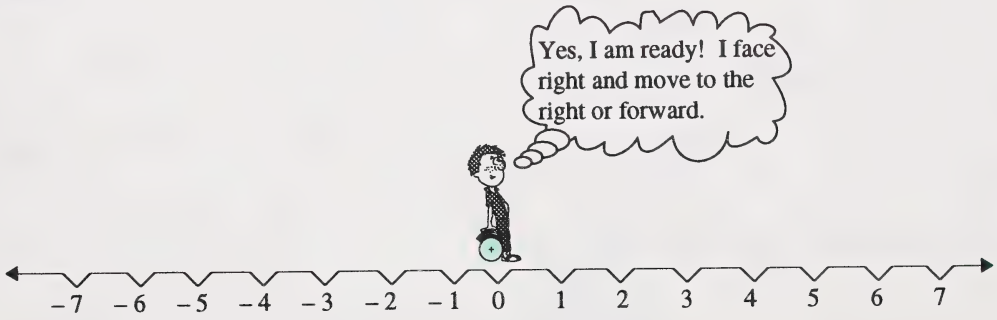
- $-3\times(+2)=-6$
- $+3\times(-2)=-6$
- $-6\div(+2)=-3$
- $+6\div(-2)=-3$

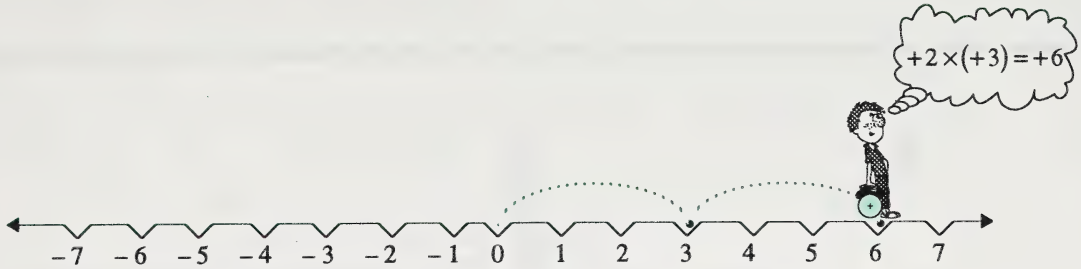
The boy with the ball is Zondex. Keep the following points in mind as you study each number line.

- Zondex always starts his bounces and movements at zero on the number line.
- He has two balls, one with a plus sign and the other with a negative sign.
- When Zondex bounces the positive ball, he faces to the right which is a positive direction.
- When Zondex bounces the negative ball, he faces to the left which is a negative direction.
- In a multiplication question, the sign of the first factor tells Zondex which ball to bounce which in turn tells him which way to face. A positive ball means face right while a negative ball means face left.
- The sign of the second factor tells Zondex which way to bounce the ball. If the sign is positive he bounces the ball as he moves forward. If the sign of the second factor is negative he bounces the ball as he moves backwards.

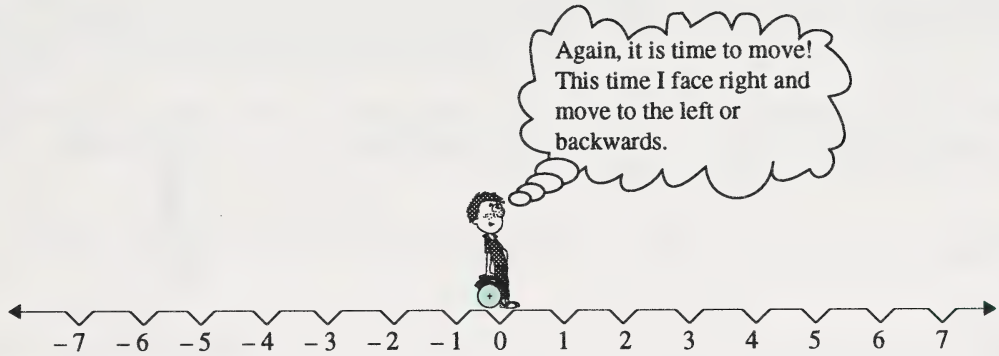
Notice what happens when Zondex is put to work.

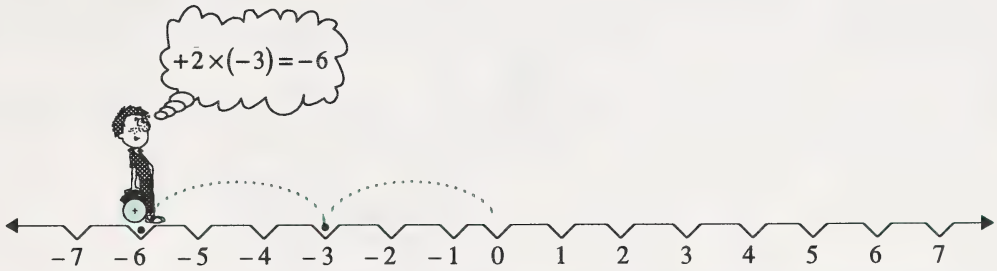
- $(+2)(+3)$ or $+2\times(+3)$



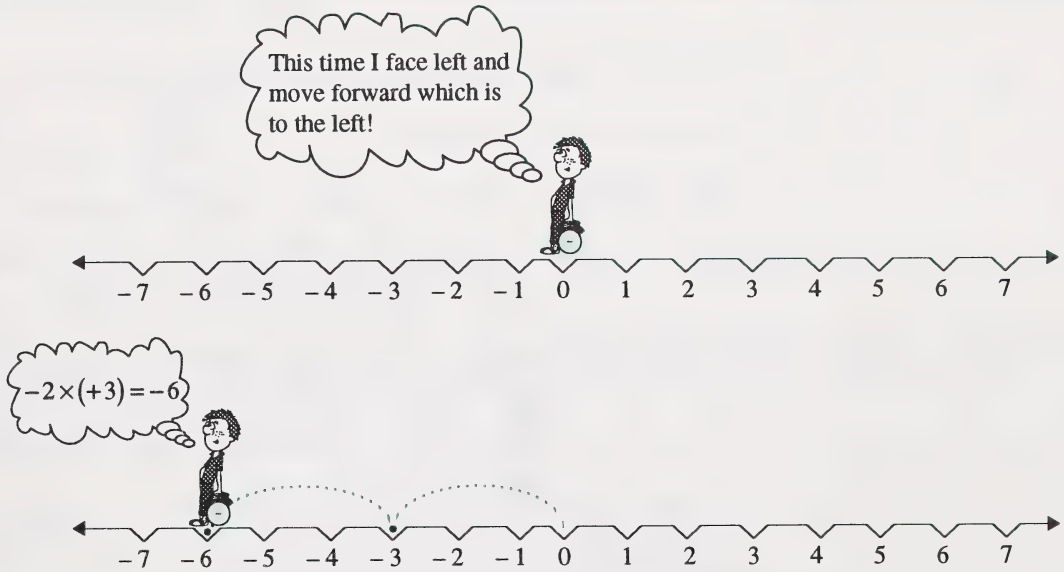


- $(+2)(-3)$ or $+2\times(-3)$

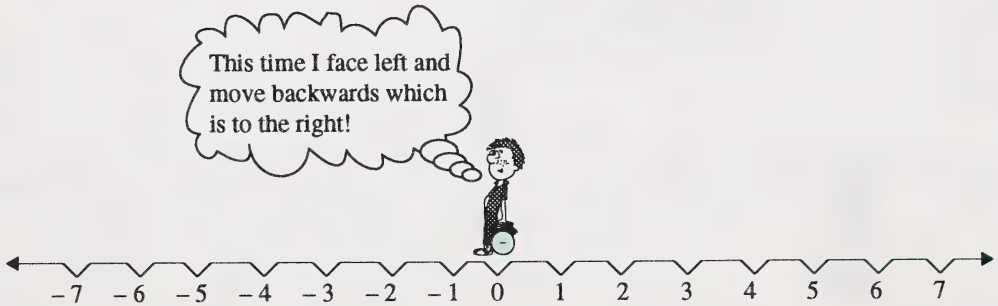


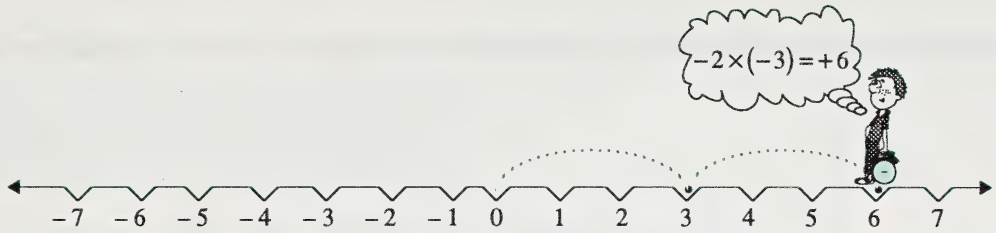


- $(-2)(+3)$ or $-2\times(+3)$



- $(-2)(-3)$ or $-2\times(-3)$





Thanks to Zondex you should have a better idea as to why some products are positive and why some products are negative.

You will recall from previous studies that the operations of multiplication and division are inverse or opposite operations. What this means is that any multiplication of two factors with its product can be changed to a division of two values with its quotient. The opposite is also true. Any division set up with its quotient can be changed to a multiplication of two factors with its product.

Zondex has shown you why you get some positive and some negative products when multiplying integers. Since multiplication can be changed to division, the same rules apply no matter which operation is used. Study the conversions which come next.

- $+2\times(+3)=+6 \rightarrow +6\div(+2)=+3$ or $+6\div(+3)=+2$
  When two positive factors are multiplied, the product is positive; and when two positive values are divided, the quotient is positive.

- $+2\times(-3)=-6 \rightarrow -6\div(+2)=-3$
  When a positive and a negative are multiplied, the product is negative; and when a positive and a negative are divided, the quotient is negative. If -3 is used as a divisor you get $-6\div(-3)=+2$ which shows that when two negative values are divided, the quotient is positive which is the same rule as for the multiplication of two negative factors.

Do the following exercises using the rules for multiplying and dividing integers.

Do any five of the eight questions.

1. Simplify.

   a. $-24+(-6)$ b. $-32+(+8)$

   c. $+32+(-6)$

2. Simplify.

   a. $-24-(+4)$ b. $+24-(-4)$

   c. $-32-(+6)$

3. Simplify.

   a. $-32\times(-3)$ b. $+32\times(-3)$

   c. $+32\times(+4)$

4. Simplify.

   a. $-16\div(-2)$ b. $+16\div(+8)$

   c. $-16\div(+4)$

5. Simplify.

   a. $-25+(+36)$ b. $+25-(-36)$

   c. $-40\times(+5)$

6. Simplify.

   a. $-40\div(-5)$ b. $+40\times(-3)$

   c. $-8-(+2)$

7. Simplify.

   a. $-64-(-20)$ b. $-64+(-20)$

   c. $-64\div(+2)$

8. a. Bill wrote a cheque to his friend for $40.00. If Bill only had $24.00 in his chequing account, by how much would his account be overdrawn?

   b. The Griffiths Giants football team had the ball on their 40 yard line. In the next play, they lost 8 yards. Where should the ball be placed for the next play? Show the equation as addition, not subtraction.

   c. The temperature at 6:00 a.m. was 6°C. By 10:00 p.m. that same day, the temperature dropped 20°C. What was the temperature at 10:00 p.m.?

For solutions to **Activity 10**, turn to the **Appendix**, **Topic 1**.

Activity 11

Perform calculations involving rational numbers.

In activities 2 through 5 you learned how to calculate with fractions and decimals. In Activity 10 you worked on calculations with integers. Now you should be ready to work on calculations with rational numbers.

When calculating with rational numbers use the signs beside the numbers to tell you what to do just as you did when working with integers. You then perform the operations as you would with fractions and decimals.

Example 23

- Solve $\frac{3}{4}+\left(\frac{-1}{2}\right)$.

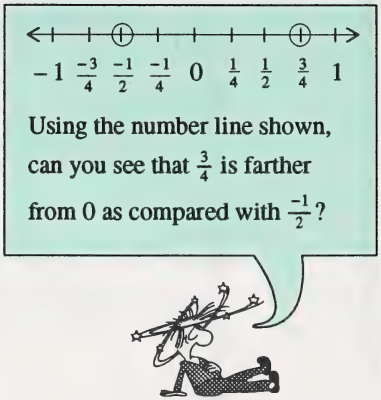
Solution:

Rule 2 for adding integers says that we need to subtract the numbers when they have different signs.

$$\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}$$
$$=\frac{1}{4}$$

Rule 2 also says to attach the sign of the number farthest from the zero to the answer. In this case $\frac{+3}{4}$ is farther from zero than is $\frac{-1}{2}$. Therefore use the positive sign in your answer.

$$\frac{3}{4}+\left(\frac{-1}{2}\right)=+\frac{1}{4} \text{ or } \frac{1}{4}$$



- Solve $\frac{-3}{4}\times\left(-2\frac{3}{4}\right)$.

Solution:

Rule 1 for multiplying and dividing integers says that the answer is positive when the signs are the same.

$$\frac{-3}{4}\times\left(-2\frac{3}{4}\right)=\frac{-3}{4}\times\left(\frac{-11}{4}\right)$$
$$=+\frac{33}{16}$$
$$=+2\frac{1}{16}$$

Because $\frac{-3}{4}$ and $-2\frac{3}{4}$ are both negative, the answer will be positive.

$$\therefore\frac{-3}{4}\times\left(-2\frac{3}{4}\right)=+2\frac{1}{16}$$

- Solve $-6-\left(+2\frac{3}{4}\right)$.

Solution:

The rule for subtracting integers asks you to change the subtraction sign to addition and to change the sign of the second number or the number being subtracted.

See how $-6-\left(+2\frac{3}{4}\right)$. becomes $-6+\left(-2\frac{3}{4}\right)$.

You now have an addition question. Rule 1 for adding integers says to add the numbers and attach the common sign as in $6+2\frac{3}{4}=8\frac{3}{4}$.

$$-6-\left(+2\frac{3}{4}\right)=-6+\left(-2\frac{3}{4}\right)$$
$$=-8\frac{3}{4}$$

- Solve $-2.4\div(+0.6)$.

Solution:

Rule 2 for multiplying and dividing integers says that the answer is negative when the signs are different.

$$0.6\overline{)2.4}=6\overline{)24}^{\,4}$$
$$\underline{24}$$
$$0$$

Note that the decimal point is moved one place to the right in both the divisor and the dividend.

Because – 2.4 and + 0.6 have different signs, the answer will be negative.

$$\therefore -2.4\div(+0.6)=-4$$

• Solve $-4+(+3.2)$.

Solution:

Rule 2 for adding integers says to subtract the numbers when the signs are different.

$$\begin{array}{r} 4 = \ 4.0 \\ \underline{-3.2} = \underline{-3.2} \\ 0.8 \end{array}$$

Rule 2 also says to attach the sign with the number farthest from zero. Since -4 is farther from zero than -3.2 is, use the negative sign in your answer. Therefore, $-4+(+3.2)=-0.8$.

Do any three of the five questions which come next.

1. Simplify.

   a. $-\frac{7}{8}+\left(\frac{+3}{4}\right)$ b. $+\frac{3}{4}\times\left(-1\frac{1}{2}\right)$

   c. $-6+\left(+1\frac{1}{4}\right)$

2. Simplify.

   a. $+\frac{7}{8}+\left(\frac{-3}{4}\right)$ b. $-\frac{3}{4}\times\left(+1\frac{1}{2}\right)$

   c. $+6+\left(-1\frac{1}{4}\right)$

3. Simplify.

   a. $-6.2+(-5)$ b. $+6.2-(-5)$

   c. $-4\times(+3.8)$

4. Simplify.

   a. $-4.2+(+2)$ b. $+5.08\times(-0.2)$

   c. $6-(-2.2)$

5. Naomi was paid \$65.00 and deposited the cheque into her chequing account. If she previously had an overdrawn account showing $-\$6.32$, how much would she now have in her account?

6. Edie has six apple pies and decides to give $1\frac{1}{2}$ pies to her friend. How many does she have left? Show the equation as addition, not subtraction.

For solutions to **Activity 11**, turn to the **Appendix, Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

For another view of the work you did with adding and subtracting fractions, you should consider this alternative.

Find the L.C.D. mathematically as shown. When you have found the L.C.D., proceed as before.

Look at these examples.

Example 24

- Solve $\frac{3}{4}+\frac{5}{6}$.

Solution:

$$\frac{3}{4}+\frac{5}{6}=\frac{3\times3}{4\times3}+\frac{5\times2}{6\times2}$$ (See how the L.C.D. was found.)

$$=\frac{9}{12}+\frac{10}{12}$$

$$=\frac{9+10}{12}$$

$$=\frac{19}{12}=1\frac{7}{12}$$

- Solve $\frac{6}{7}-\frac{2}{5}$.

Solution:

$$\frac{6}{7}-\frac{2}{5}=\frac{6\times5}{7\times5}-\frac{2\times7}{5\times7}$$

$$=\frac{30-14}{35}$$

$$=\frac{16}{35}$$

- Solve $1\frac{1}{3}-\frac{8}{9}$.

Solution:

$$1\frac{1}{3}-\frac{8}{9}=\frac{4}{3}-\frac{8}{9}$$ (Change $1\frac{1}{3}$ to an improper fraction.)

$$=\frac{4\times3}{3\times3}-\frac{8\times1}{9\times1}$$ (Proceed as before.)

$$=\frac{12}{9}-\frac{8}{9}$$

$$=\frac{12-8}{9}$$

$$=\frac{4}{9}$$

When you have mixed numbers to add and subtract, you may take all the whole number parts and treat them as a unit; then, do the same with the fractional parts. This is often called the separation method. The following example illustrates this.

Example 25

- Solve $9\frac{1}{5}+3\frac{1}{3}$.

  Solution:

$$9\frac{1}{5}+3\frac{1}{3}=(9+3)+\frac{1}{5}+\frac{1}{3}$$
$$=12+\frac{3+5}{15}$$
$$=12+\frac{8}{15}$$
$$=12\frac{8}{15}$$

- Solve $7\frac{2}{5}-3\frac{2}{7}$.

  Solution:

$$7\frac{2}{5}-3\frac{2}{7}=(7-3)+\left(\frac{2}{5}-\frac{2}{7}\right)$$
$$=4+\frac{14-10}{35}$$
$$=4+\frac{4}{35}$$
$$=4\frac{4}{35}$$

- Solve $3\frac{1}{2}-4\frac{1}{3}+2\frac{2}{3}$.

  Solution:

$$3\frac{1}{2}-4\frac{1}{3}+2\frac{2}{3}=(3-4+2)+\left(\frac{1}{2}-\frac{1}{3}+\frac{2}{3}\right)$$
$$=(5-4)+\left(\frac{3-2+4}{6}\right)$$
$$=1+\frac{7-2}{6}$$
$$=1+\frac{5}{6}$$
$$=1\frac{5}{6}$$

You should note that there are several ways of adding and subtracting rational numbers. You may use the one that is best suited to you. No one method is the best.

When you do algebra, you will work with rational numbers again. You will find that if you remember and understand what you have learned in arithmetic, it will make your next task so much easier.

So often you will find that one concept is only an extension of a previous one, and if you fully understood the first one, then the next will be quite easy to grasp.

In this **Extra Help** section, you will learn one more technique for multiplying fractions and mixed numbers. Be sure to follow the steps, as they are very important to the whole procedure.

When you multiply rational numbers that involve fractions, you may use the process of **eliminating factors**. This reduces and simplifies the fractions to a basic form. Some texts refer to this process as the method of **cancelling**. Try an example.

Example 26

- Find the product of $\frac{9}{15}\times\frac{8}{12}$.

Solution:

Divide the top and bottom by a common factor, in this case, 3.

$$\frac{\overset{3}{\cancel{9}}}{15}\times\frac{8}{\underset{4}{\cancel{12}}}$$

Divide the top and bottom by another common factor, 4.

$$\frac{3}{15}\times\frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}}}$$

Divide the top and bottom by another common factor, 3.

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{15}}}\times\frac{2}{1}$$

Continue until there are no common factors except 1 remaining.

$$\frac{1}{5}\times\frac{2}{1}=\frac{1\times2}{5\times1}$$

$$=\frac{2}{5}$$

The answer is $\frac{2}{5}$.

- Find the product of $2\frac{1}{4}\times1\frac{1}{3}$.

Solution:

$$2\frac{1}{4}\times1\frac{1}{3}=\frac{\overset{3}{\cancel{9}}}{4}\times\frac{4}{\underset{1}{\cancel{3}}}$$

$$=\frac{3}{\underset{1}{\cancel{4}}}\times\frac{\overset{1}{\cancel{4}}}{1}$$

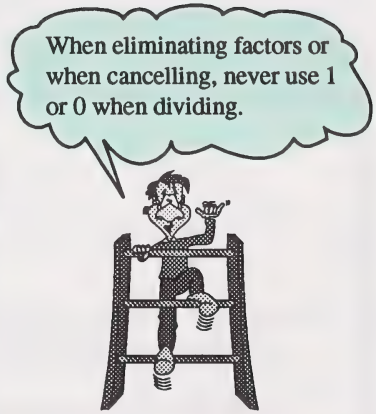
$$=\frac{3}{1}\times\frac{1}{1}$$

$$=\frac{3\times1}{1\times1}$$

$$=\frac{3}{1}$$

$$=3$$

The answer is 3.



Eliminating factors or cancelling helps by

- making multiplication of fractions easier
- making multiplication of fractions quicker

Do any four of the following.

1. Add the following.

   a. $\frac{5}{6}+\frac{2}{3}$

   b. $\frac{7}{9}+\frac{3}{4}$

   c. $11\frac{7}{8}-6\frac{5}{6}$

2. Subtract the following.

   a. $\frac{5}{8}-\frac{7}{12}$

   b. $9\frac{1}{8}-6\frac{7}{8}$

   c. $10\frac{3}{8}-7\frac{1}{9}$

3. Evaluate by using the separation method.

   a. $1\frac{1}{2}+2\frac{2}{3}+1\frac{1}{4}$

   b. $9\frac{1}{2}-11\frac{3}{4}+5\frac{1}{3}$

   c. $7\frac{3}{4}-2\frac{1}{8}-3\frac{1}{3}$

4. Simplify the following by cancelling.

   a. $\frac{2}{3}\times\frac{8}{16}\times\frac{9}{5}$

   b. $\frac{5}{6}\times\frac{7}{15}\times\frac{18}{21}$

   c. $1\frac{1}{2}\times2\frac{2}{9}\times3\frac{3}{10}$

5. Calculate the following.

   a. $1\frac{1}{3}-3\frac{1}{2}$

   b. $5\frac{1}{4}+3\frac{1}{5}$

   c. $-1\frac{1}{8}+2\frac{1}{5}$

   d. $-2\frac{5}{12}-3\frac{1}{7}$

   e. $-7\frac{1}{3}+3\frac{1}{7}$

   f. $2\frac{5}{6}-1\frac{7}{8}$

6. Calculate the following.

   a. $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}$

   b. $2\frac{1}{5}-3\frac{1}{4}+2\frac{1}{3}$

   c. $5\frac{8}{9}+13\frac{1}{6}$

   d. $3\frac{1}{17}-4\frac{2}{13}$

For solutions to **Extra Help**, turn to the **Appendix**, **Topic 1**.

Extensions

Many scientific instruments have scales which must be read in decimal values with the last place estimated and rounded.

To extend your knowledge and understanding of rounding and estimating with decimals, you may use the Apple II software program entitled *Mathematics for Science, Series 1: Measurement* [1].

[1] *Mathematics for Science, Series 1, Measurement* is a title of Merlan Scientific Ltd.

Multiplying and Dividing by Powers of 10

When you multiply or divide a number by powers of 10, you can arrive at the answer by simply moving the decimal point to the right when multiplying or to the left when dividing.

Look at the following examples.

Example 27

- Compute 183.45×10.

  Solution:

  $183.45 \times 10 = 1834.5$

  Move the decimal **one** place to the right since you are multiplying by 10 or 10^1.

- Compute 0.152×100.

  Solution:

  $0.152 \times 100 = 15.2$

  Move the decimal **two** places to the right since you are multiplying by 100 or 10^2.

- Compute 400×0.021.

  Solution:

  $$400 \times 0.021 = (4 \times 100) \times 0.021$$
  $$= 4 \times (100 \times 0.021)$$
  $$= 4 \times 2.1$$
  $$= 8.4$$

Example 28

- Compute $484.01 \div 100$.

  Solution:

  $$484.01 \div 100 = 4.8401$$

  You move the decimal **two** places to the left since you are dividing by 100 or 10^2.

- Compute $0.014 \div 10$.

  Solution:

  $$0.014 \div 10 = 0.0014$$

  You move the decimal **one** place to the left since you are dividing by 10 or 10^1.

- Compute $48.4 \div 1000$.

  Solution:

  $$48.4 \div 1000 = 0.0484$$

  You move the decimal **three** places to the left since you are dividing by 1000 or 10^3.

Did you notice that the 3 in 10^3 tells how many places the decimal point is moved to the left?

1. Move a decimal point to multiply each of these.

   a. 1.86×10 b. 343×100

   c. 0.014×100 d. 1000×0.84

   e. $0.0001 \times 10\,000$

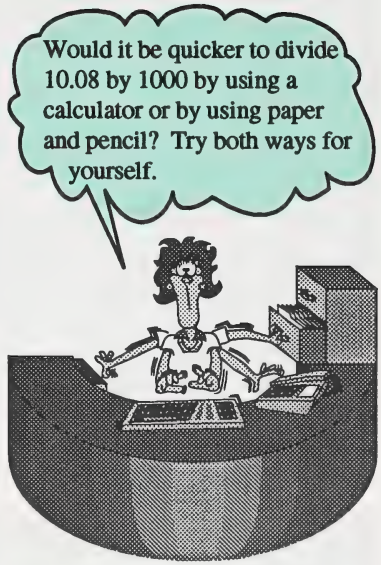
2. Find the quotient for each problem by moving a decimal point.

   a. $34.3 \div 100$ b. $0.17 \div 10$

   c. $10.01 \div 1000$ d. $45 \div 1000$

   e. $0.0098 \div 10$

For solutions to **Extensions** turn to the **Appendix, Topic 1**.



Topic 2 Percent

Introduction

If you browse through most of the popular magazines or newspapers, you will see that many of the contributors make extensive use of percent.

Look at the classified advertising and the financial pages. You will notice that governments, realtors, salespeople, consumers, and sportswriters often use percent to provide information.

You will learn quite a bit about percent in this topic. You, should be able to use percentage to understand such material or to accurately use percentage to present your own ideas and related figures.

What Lies Ahead

Throughout the topic you will learn to

1. change a fraction to a percent and vice versa
2. change a decimal to a percent and vice versa
3. make calculations using percents
4. solve problems involving percents

Now that you know what to expect, turn the page to begin your study of percent.

Exploring Topic 2

Activity 1

Change a fraction to a percent and vice versa.

If you scan through any business or financial paper, you will notice the frequent use of the percent symbol %.

Percent is a special ratio which literally means **out of 100.**

Mathematically speaking, when you say 12% of the people are unemployed, what you mean is that 12 out of every 100 people are out of work, or you may say $\frac{12}{100}$ of the people are unemployed.

You will now learn how to change fractions to percents.

Study the following examples which show how to change a fraction to a percent.

Example 1

- Write $\frac{1}{4}$ as a percent.

Solution:

$$\frac{1}{4} = \frac{1\times 25}{4\times 25}$$
$$= \frac{25}{100}$$
$$= 25\%$$

Some fractions have a denominator that can easily be changed to 100. Some common ones would be 2, 4, 5, 10, 20, 25, and 50.

- Write $\frac{5}{6}$ as a percent.

Solution:

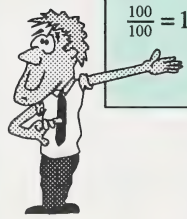
Let $\frac{5}{6} = \frac{n}{100}$ (Compare the fraction with $\frac{n}{100}$.)

$\frac{5\times 100}{6} = \frac{n\times 100}{100}$ (Multiply both sides by 100.)

$n = \frac{500}{6}$

$n = 83\frac{1}{3}$

Therefore, $\frac{5}{6} = 83\frac{1}{3}\%$, $83.\overline{3}\%$, or $83.\dot{3}\%$.



In $\frac{n\times 100}{100}$ you are really multiplying by 1 since $\frac{100}{100} = 1$.

- Write $\frac{5}{12}$ as a percent.

Solution:

$$\text{Let } \frac{n}{100} = \frac{5}{12}$$

$$\frac{n \times 100}{100} = \frac{5 \times 100}{12}$$

$$n = \frac{500}{12}$$

$$= \frac{125}{3}$$

$$= 41\frac{2}{3}$$

Therefore, $\frac{5}{12} = 41\frac{2}{3}\%$, $41.\overline{6}\%$, or $41.\dot{6}\%$.

Take the same approach if you have an improper fraction which is to be changed to a percent.

Example 2

Express $\frac{5}{4}$ as a percent.

Solution:

$$\text{Let } \frac{n}{100} = \frac{5}{4}$$

$$\frac{n \times 100}{100} = \frac{5 \times 100}{4}$$

$$n \times \frac{100}{100} = 5 \times \frac{100}{4}$$

$$n = 5 \times 25$$

$$= 125$$

$$\therefore \frac{5}{4} = 125\%$$

The reverse operation is also possible. You can write a percent as a fraction. In order to do this, simply divide the percent by 100 and simplify by eliminating common factors.

Example 3

- Write 45% as a fraction in simplest form.

Solution:

$$45\% = \frac{45}{100}$$
$$= \frac{9}{20}$$

- Write $66\frac{2}{3}\%$ as a basic fraction.

Solution:

$$66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100}$$
$$= \frac{\frac{200}{3}}{100}$$
$$= \frac{200}{3} \div \frac{100}{1}$$
$$= \frac{200}{3} \times \frac{1}{100}$$
$$= \frac{200}{300}$$
$$= \frac{2}{3}$$

- Write 1.5% as a fraction in simplest form.

Solution:

$$1.5\% = \frac{1.5}{100}$$
$$= \frac{1.5 \times 10}{100 \times 10}$$
$$= \frac{15}{1000}$$
$$= \frac{3}{200}$$

Do any three of questions 1 to 5. Then complete questions 6 to 8.
Write each of the following as percents.

1.	a. $\frac{1}{5}$	b. $\frac{3}{4}$	c. $\frac{4}{5}$
2.	a. $\frac{1}{3}$	b. $\frac{5}{6}$	c. $\frac{2}{15}$
3.	a. $\frac{4}{25}$	b. $\frac{19}{20}$	c. $\frac{5}{12}$
4.	a. $1\frac{1}{2}$	b. $1\frac{1}{6}$	c. $2\frac{2}{3}$
5.	a. $\frac{1}{20}$	b. $1\frac{1}{50}$	c. $\frac{17}{20}$

6. If three people out of twenty were unemployed, what percent of the work force was out of work?

7. In an aerobics class, there were 55 students, and 30 were girls. What percent of the class were boys? Round your answer to the nearest whole percent.

8. Write the following percents as common fractions.

   a. 75% b. 35%

   c. 5% d. $12\frac{1}{2}\%$

   e. $33\frac{1}{3}\%$ f. $16\frac{2}{3}\%$

For solutions to **Activity 1**, turn to the **Appendix**, **Topic 2**.

Activity 2

Change a decimal to a percent and vice versa.

You will find it necessary to change a decimal number to a percent in your everyday activities. Statisticians and accountants work a lot with decimal numbers and percents.

In order to write a decimal number as a percent, move the decimal point two places to the right and attach the percent symbol. The percent symbol is %.

In order to write a percent as a decimal number, move the decimal point two places to the left and remove the percent symbol. The percent symbol is %.

Example 4

- Write 0.12 as a percent.

  Solution:

  $0.12 = 12\%$

  The decimal point was moved two places to the right.

- Write 0.025 as a percent.

  Solution:

  $0.025 = 2.5\%$

  The decimal point in the percent is very necessary.

- Write 0.7 as a percent.

  Solution:

  $0.7 = 70\%$

  0.7 becomes 0.70 so you can now move the decimal point two places to the right.

- Write 2.35 as a percent.

  Solution:

  $2.35 = 235\%$

Since a percent is a value out of 100, when you change a percent to a decimal number, divide by 100. This means moving the decimal point two places to the left. See how this is done in the example which follows.

Example 5

- Write 35% as a decimal.

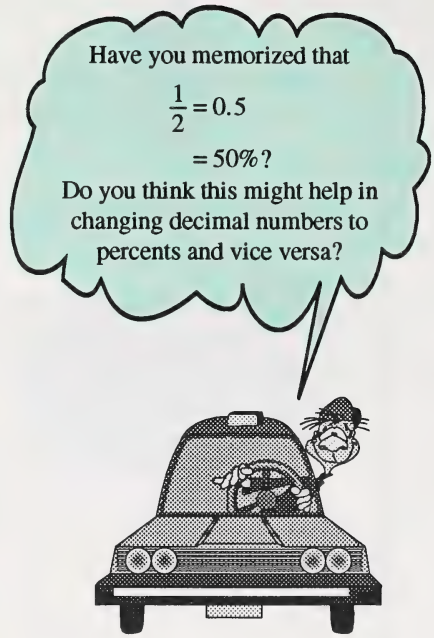
  Solution:

$$35\% = \frac{35}{100}$$
$$= 0.35$$

- Write $12\frac{1}{2}\%$ as a decimal.

  Solution:

$$12\frac{1}{2}\% = 12.5\%$$
$$= \frac{12.5}{100}$$
$$= 0.125$$



- Write 0.05% as a decimal.

  Solution:

$$0.05\% = \frac{0.05}{100}$$
$$= 0.0005$$

- Write $166\frac{2}{3}\%$ as a decimal.

  Solution:

$$166\frac{2}{3}\% = \frac{166\frac{2}{3}}{100}$$
$$= 166\frac{2}{3} \div 100$$
$$= \frac{500}{3} \times \frac{1}{100}$$
$$= \frac{500}{300}$$
$$= 1.\overline{6}$$

Try some exercises based on changing decimals to percents and vice versa.

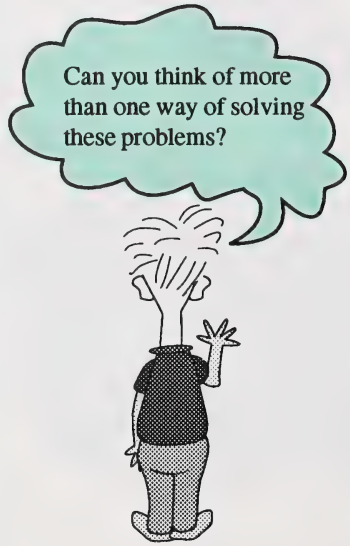
Write the following decimals as percents.

1. a. 0.6 b. 0.04
   c. 0.32 d. 0.1
2. a. 0.125 b. 0.009
   c. 0.105 d. 0.0055
3. a. 1.5 b. 2.25
   c. 1.9 d. 1.05

Write the following percents as decimals.

4. a. 6% b. 19%
   c. 2% d. 100%
5. a. $1\frac{1}{2}\%$ b. 0.5%
   c. $33\frac{1}{3}\%$ d. 275%
6. During the last civic election 0.85 of the people went to the polls. What percent did not go?
7. If 0.375 of the cargo of a plane was lost somewhere, what percent made it to safety?

For solutions to **Activity 2**, turn to the Appendix, **Topic 2**.



Activity 3

Make calculations using percents.

In order to calculate with percent, you will use previous skills developed from working with decimal numbers and fractions. You have learned how to change decimal numbers and fractions to percent and vice versa. Now, you will learn a few more skills in this section.

You have probably noticed in your local paper such expressions as

- 30% discount
- mortgage rates
- the salesperson's commission is 7%
- 10% down on furniture

There are many other questions on percent that you will meet almost everyday as well.

Pay close attention to

- what information you are given
- what information you are asked to find

Now examine a few examples.

Example 6

What percent of \$55.00 is \$35.00 rounded to the nearest whole percent?

Solution:

In this question you are asked to compare \$35.00 with \$55.00 and to express your answer as a percent rounded to the nearest whole percent.

The best way to compare two quantities is by using a ratio. The ratio in this case is 35 to 55 or 35:55.

Next, $\frac{35}{55}$ is compared to another ratio where the denominator is 100. The resulting proportion is then solved.

$$\frac{35}{55} = \frac{n}{100}$$

$$\frac{35 \times 100}{55} = \frac{n \times 100}{100}$$

$$n = \frac{3500}{55}$$

$$= 63.\overline{63}$$

$$\doteq 64$$

Therefore, \$35.00 is approximately 64% of \$55.00.

Example 7

Find 20% of 65.

Solution:

In this question you are asked to compare a number with 65 that will be equivalent to 20%.

If you call this number x, then you can write

$$\frac{x}{65} = \frac{20}{100} \quad (20\% \text{ is equivalent to } \tfrac{20}{100}.)$$

$$x \times 100 = 20 \times 65 \quad \text{(Cross multiply.)}$$

$$\frac{x \times 100}{100} = \frac{20 \times 65}{100} \quad \text{(Divide both sides by 100.)}$$

$$x = \frac{20 \times 65}{100} \quad \text{(Simplify.)}$$

$$x = \frac{1300}{100} \quad \text{(Simplify.)}$$

$$x = 13$$

Therefore, 20% of 65 is 13.

Example 8

If 35% of a quantity of money is \$18.00, what is the original quantity of money?

Solution:

Let x be the original quantity of money.

In this question, you consider \$18.00 as part of the whole quantity and compare this to 35% or $\frac{35}{100}$.

To calculate, set up the equivalent fractions.

$$\frac{18.00}{x} = \frac{35}{100}$$

$$35 \times x = 18.00 \times 100 \quad \text{(Cross multiply.)}$$

$$x = \frac{1800.00}{35} \quad \text{(Divide both sides by 35.)}$$

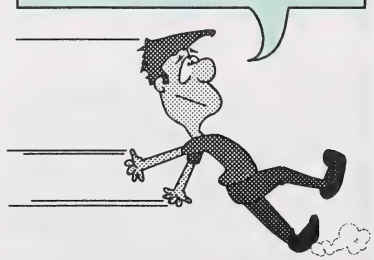
$$x = 51.43 \quad \text{(Rounded to the nearest cent.)}$$

The original quantity of money is \$51.43.

It is important to note that when you are doing calculations, you can add percents because they are all out of 100. For example, $30\% + 20\% + 10\% = 60\%$.

Stop and consider these points.

- **What percent** means $\frac{x}{100}$.
- $x\%$ means $\frac{x}{100}$.
- Whatever comes after the word **of**, goes at the bottom of the other fraction or ratio. Using $x\%$ of 37 is 15 you get $\frac{x}{100} = \frac{15}{37}$.



Now try some exercises using calculations in percent. Do any six of the following.

1. What is $\frac{84}{25}$ expressed as a percent?

2. $x\%$ of 64 is 48. What is x?

3. If 95% of a debt is \$760.00, what is the total debt?

4. What percent of 72 is 45?

5. Find 35% of \$105.00.

6. If 30% of x is 60, what is x?

7. Find the sum of 5%, 15%, 27%, and 43%.

8. Find 75% of 55.

For solutions to **Activity 3**, turn to the **Appendix**, **Topic 2**.

Activity 4

Solve problems involving percent.

If you want to solve problems using percent, you should be aware of the following.

- Understand the information you are given.
- Know what information you are asked to find.
- Know how to use the information you have.

Basically there are three types of problems involving percent. They may be worded in various ways, but they all may be expressed in the following form.

n is $x\%$ of w.

In any percent problem you will need to find one of the unknowns stated previously. Of the three values, two must be given. The three types of information you need to find may be organized as follows.

- What number is a known percent of a known whole quantity? For example, **what number** is 20% of 55 or n is 20% of 55.

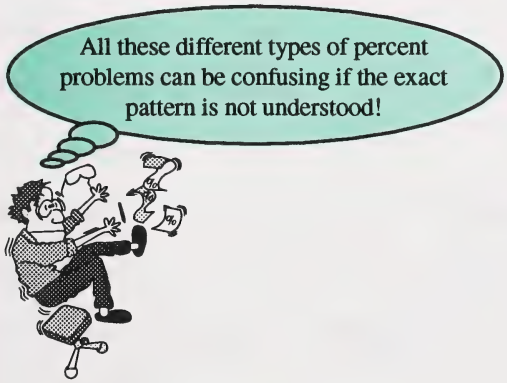
- A known number is what percent of a known whole quantity? For example, 11 is **what percent** of 55 or 11 is $x\%$ of 55.

- A known number is a known percent of what whole quantity? For example, 11 is 20% of **what number** or 11 is 20% of w.

Before you attempt a problem involving percents, you should first be able to identify which of the three types you are working with. Your decision is based on which pieces of information are given.

Most times you will be required to read a statement and interpret what you are supposed to do. Percent problems are generally expressed in words and your complete understanding of what is given and what you must find will determine whether you get the solution right or not.

Consider a few examples which show problem solving with percents.



Example 9

Cedric's basketball team won 75% of their games. If they played 16 games in all, how many did they win?

Solution:

The question is how many games did they win out of the 16 they played. This problem gives you the percent and the whole quantity. The problem becomes what number is 75% of 16? To solve this problem, you compare ratios.

$$\frac{n}{16} = \frac{75}{100}$$
$$100 \times n = 16 \times 75$$
$$\frac{100n}{100} = \frac{16 \times 75}{100}$$
$$n = \frac{4 \times 75}{25}$$
$$= 4 \times 3$$
$$= 12$$

They won 12 of the 16 games played.

Example 10

The sale price of a ten-speed bicycle is $179.98. This price is 80% of the original price. Calculate the original selling price.

Solution:

The price paid is $179.98. This is 80% of the original price. The problem may be stated as $179.98 is 80% of what original price?

Set up the equivalent ratios and solve this proportion $\frac{179.98}{n} = \frac{80}{100}$.

Use a calculator to solve the problem.

Enter	Display
179.98	179.98
×	179.98
100	100
+	17998
80	80
×	224.975

$n \doteq \$224.98$

The original price was $224.98.

Example 11

The Comets won 13 of their baseball games. If they played 20 games in all, what percent of the games did they win?

Solution:

The problem is 13 is what percent of 20?

$$\frac{13}{20} = \frac{n}{100}$$
$$20n = 13 \times 100$$
$$\frac{20x}{20} = \frac{13 \times 100}{20}$$
$$n = \frac{1300}{20}$$
$$= 65$$

They won 65% of the games played.

Whenever you solve a problem, always ask yourself if the answer you get is a **reasonable** answer. If your answer is not reasonable, review and redo your work.

Example 12

Mrs. Koy sells bottles of perfume for $6.90 each. If she earns 40% of the sale as commission, how much does she earn on each bottle that she sells?

Solution:

You are asked to find 40% of $6.90. Let the commission be n.

$$\frac{n}{6.90} = \frac{40}{100}$$

$$100n = 40 \times 6.90$$

$$\frac{100n}{100} = \frac{40 \times 6.9}{100}$$

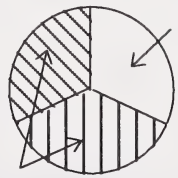
$$n = \frac{40 \times 6.9}{100}$$

$$= \frac{276}{100}$$

$$= 2.76$$

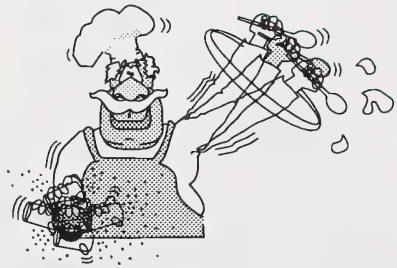
Therefore, Mrs. Koy earns $2.76 commission on each bottle of perfume that she sells.

Suppose you have a pizza and two of three equal parts are eaten. The portion eaten is $\frac{2}{3}$ or $66\frac{2}{3}\%$ of the whole pizza. The portion not eaten is $\frac{1}{3}$ or $33\frac{1}{3}\%$ of the whole pizza. Other fractions and their equivalent percents can be similarly compared.



Not eaten portion is $\frac{1}{3}$ or $33\frac{1}{3}\%$ of the whole pizza.

Eaten portion is $\frac{2}{3}$ or $66\frac{2}{3}\%$ of the whole pizza.



These are the steps for solving word problems!

- Understand the problem.
- Be sure you are clear on what is given and what is asked for.
- Look for hidden questions.
- Estimate a reasonable answer.
- Set up equations and solve them.
- Double check your answer.
- Clearly state your answer in a sentence.

Do any five of the following problems.

1. In Miss Pelkie's class, there are 35 students. If 21 of the students are boys, what percent of the class are girls?

2. Milton got 72 problems right on a test with 90 problems. What percent of the problems were right?

3. A machine is performing at 70% of capacity. If capacity is 420 cases output per day, how many cases are actually being produced each day?

4. Of 240 girls polled, 160 preferred coffee to tea. What percent of the girls preferred tea?

5. Of 240 hamburgers sold in the cafeteria, $66\frac{2}{3}\%$ were cheeseburgers. How many were plain hamburgers?
   (Hint: $100\% = 99\frac{3}{3}\%$ and
   $99\frac{3}{3}\% - 66\frac{2}{3}\% = 33\frac{1}{3}\%$.)

6. The attendance at a college on a certain day was 840. If this was 80% of the total enrolment, what was the enrolment at the college?

7. A camper was sold for \$16 900.00. If the down payment was $12\frac{1}{2}\%$ of the price, what was the amount of the down payment?

8. What should Alana sell her house for in order to realize a profit of 9% on the purchase price if she paid \$49 000.00 for the house?

For solutions to **Activity 4**, turn to the **Appendix, Topic 2**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Percentages

Definition of Percent

Percent means **per hundred**. A percent is a ratio with a denominator of 100. Use the symbol %.

For example,

$$25 \text{ percent} = \frac{25}{100} = 25\%$$

$$315 \text{ percent} = \frac{315}{100} = 315\%$$

$$100 \text{ percent} = \frac{100}{100} = 100\%$$

Changing Fractions and Decimal Numbers to Percents

There are two ways of changing a fraction to a percent.

Method 1

Use a proportion with a variable for the unknown percent.

Example 13

Convert $\frac{5}{8}$ to a percent.

Solution:

$$\frac{5}{8} = \frac{x}{100}$$

$$8x = 500$$

$$\frac{8x}{8} = \frac{500}{8}$$

$$x = 62.5$$

$$\therefore \frac{5}{8} = \frac{62.5}{100}$$

$$= 62.5\%$$

Method 2

Change the given fraction to a decimal number. Then change the decimal number to a percent. Move ahead to Example 14 which shows you how Method 2 is applied.

Example 14

Convert $\frac{5}{8}$ to a percent by first changing $\frac{5}{8}$ to a decimal number.

Solution:

$$\frac{5}{8} = 0.625$$
$$= 0.625 \times 100$$
$$= 62.5\%$$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

Thus, $\frac{5}{8} = 62.5\%$.

Note that when changing a decimal number to a percent, move the decimal point two places to the right and added a % symbol after the resulting number.

Changing a Percent to a Fraction or a Decimal Number

To change a percent to a fraction, write the percent in ratio form with a denominator of 100 then reduce the fraction. To change a percent to a decimal, move the decimal point two places to the left and drop the % symbol.

Example 15

- Change 75% to a fraction and to a decimal number.

Solution:

$$\text{The fraction for } 75\% = \frac{75}{100}$$
$$= \frac{3}{4}$$

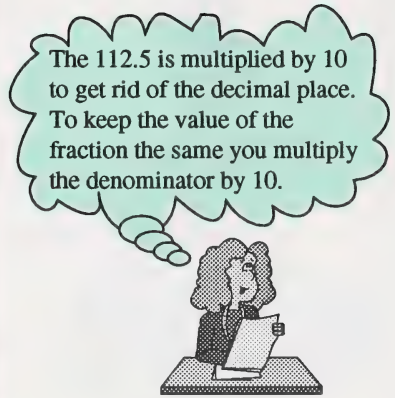
$$\text{The decimal number for } 75\% = \frac{75}{100}$$
$$= 0.75$$

- Change 112.5% to a fraction and to a decimal number.

Solution:

$$\text{The fraction for } 112.5\% = \frac{112.5}{100}$$
$$= \frac{1125}{1000}$$
$$= 1\frac{125}{1000}$$
$$= 1\frac{1}{8}$$

$$\text{The decimal number for } 112.5\% = \frac{112.5}{100}$$
$$= 1.125$$


The 112.5 is multiplied by 10 to get rid of the decimal place. To keep the value of the fraction the same you multiply the denominator by 10.

Setting up Percentage Problems

Example 16

- 6% of what number is 36?

  Solution:

  6% means $\frac{6}{100}$. Whatever comes after the word **of**, goes at the bottom of the second fraction. Whatever is left goes at the top of the second fraction.

  $$\frac{6}{100} = \frac{36}{x}$$
  $$6x = 100 \times 36$$
  $$\frac{6x}{6} = \frac{3600}{6}$$
  $$x = 600$$

  Therefore, 6% of 600 is 36.

- 2% of what number is 8?

  Solution:

  Use $\frac{2}{100} = \frac{8}{x}$ since 2% means $\frac{2}{100}$. **What number** comes after the word of; so the x goes at the bottom of the second fraction. 8 is left and is used as the numerator of the second ratio.

  $$2x = 8 \times 100$$
  $$\frac{2x}{2} = \frac{800}{2}$$
  $$x = 400$$

  Therefore, 2% of 400 is 8.

Example 17

- 6% of 600 is what number?

  Solution:

  $$\frac{6}{100} = \frac{x}{600}$$
  $$100x = 6 \times 600$$
  $$100x = \frac{3600}{100}$$
  $$x = 36$$

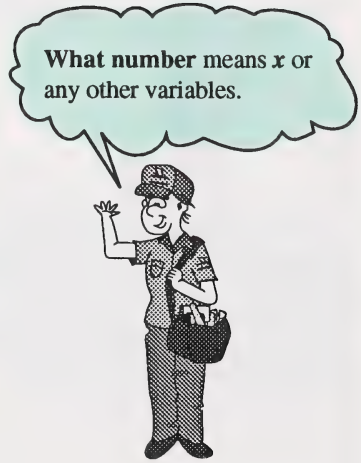
  Therefore, 6% of 600 is 36.

- 2% of 400 is what number?

  Solution:

  $$\frac{2}{100} = \frac{x}{400}$$
  $$100x = 2 \times 400$$
  $$\frac{100x}{100} = \frac{800}{100}$$
  $$x = 8$$

  Therefore, 2% of 400 is 8.



Example 18

- What percent of 600 is 36?

Solution:

$$\frac{x}{100} = \frac{36}{600}$$
$$600x = 36 \times 100$$
$$\frac{600x}{600} = \frac{3600}{600}$$
$$x = 6$$

Therefore, 6% of 600 is 36.

- What percent of 400 is 8?

Solution:

$$\frac{x}{100} = \frac{8}{400}$$
$$400x = 8 \times 100$$
$$\frac{400x}{400} = \frac{800}{400}$$
$$x = 2$$

Therefore, 2% of 400 is 8.

Do either question 1 or question 2, and all of questions 3, 4, and 5.

1. Change each fraction to a percent.

   a. $\frac{3}{8}$ b. $\frac{3}{4}$ c. $\frac{5}{6}$ d. $\frac{13}{10}$

2. Change each fraction to a decimal numeral and to a percent.

   a. $\frac{1}{8}$ b. $\frac{4}{3}$ c. $\frac{9}{10}$ d. $\frac{3}{5}$

3. Change each percent to a fraction and to a decimal numeral.

   a. 25% b. 45%

   c. 116.25% d. $33\frac{1}{3}\%$

   e. 30% f. $16\frac{2}{3}\%$

4. Solve for n in each of the following.

   a. 40% of 75 = n b. 63% of 1156 = n

   c. n% of 90 = 25 d. n% of 135 = 60

   e. 17% of n = 42 f. 39.5% of n = 1034

5. A student forgot to return a library book. The library fine is 5¢/day per overdue book, or 85% of the list price for lost books. If the list price of the book is $9.00, should the student pay the fine after 170 days or purchase the overdue book? (Show all your calculations.)

For solutions to **Extra Help**, turn to the **Appendix**, **Topic 2**.

Extensions

Percents are used to quote discounts, interest rates, efficiency and so on. Related to these quotes are millrates and parts-per-million.

A millrate is used as a basis for calculating property taxes. A mill is $\frac{1}{1000}$ of a dollar. It can also be expressed as $\frac{1}{10}$ of one cent. The $\frac{1}{1000}$ of a dollar and the $\frac{1}{10}$ of a cent can be expressed as percents.

Parts-per-million is used in chemistry. Many times it is used to express the toxic potential of a drug or medicine. Dioxyn or PCB's are highly toxic chemicals. Small parts-per-million are lethal to most organisms. These too can be expressed as a percent.

Example 19

It is known that 90 parts-per-million of a chemical will kill all bacteria but not fish. Express this as a decimal and a percent.

Solution:

$$90 \text{ parts-per-million} = \frac{90}{1\,000\,000}$$

Expressed as a decimal it equals 0.000 09.

Expressed as a percent it equals $0.000\,09 \times 100 = 0.009\%$.

Example 20

- Express a mill as a percent.

Solution:

$$\frac{1}{1000} = \frac{n}{100}$$

$$1000n = 1 \times 100$$

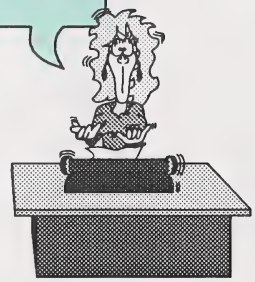
$$\frac{1000n}{1000} = \frac{100}{100}$$

$$n = \frac{1}{10}$$

Therefore, $\frac{1}{1000} = \frac{1}{10}\%$ or 0.1%, and

1 mill $= \frac{1}{10}\%$ or 0.1%.

To change 0.000 09 to a percent, the decimal point is moved two places to the right to get 0.009%. Moving the decimal point two places to the right is the same as multiplying by 100.



- Express 100 mills as a percent.

Solution:

$$\frac{100}{1000} = \frac{1}{10}$$

$$\frac{1}{10} = \frac{n}{100}$$

$$10n = 1 \times 100$$

$$\frac{10n}{10} = \frac{100}{10}$$

$$n = 10$$

Therefore, $\frac{1}{10}$ or $\frac{100}{1000} = 10\%$, and 100 mills $= 10\%$.

- Express 1000 mills as a percent.

Solution:

$$\frac{1000}{1000} = 1 \text{ and}$$

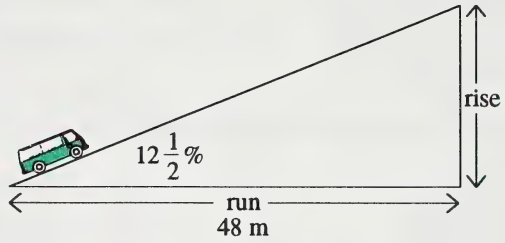
$$1 \times 100 = 100\%$$

Therefore, 1000 mills $= 100\%$.

A slope or gradient is sometimes expressed in terms of a percent.

Example 21

How high is a hill if its slope is $12\frac{1}{2}\%$ from a run of 48 metres?



Solution:

$$\text{The slope of the hill} = 12.5\%$$

$$= 0.125$$

$$= \frac{125}{1000}$$

$$= \frac{1}{8}$$

Let the height of hill be x m.

The slope $= \frac{\text{rise}}{\text{run}}$ which results in $\frac{x}{48}$.

$$\therefore \frac{1}{8} = \frac{x}{48}$$

$$\frac{1}{8} = \frac{x}{48} \quad \text{(Solve for } x\text{.)}$$

$$8x = 1 \times 48$$

$$\frac{8x}{8} = \frac{48}{8}$$

$$x = 6$$

The hill is 6 m high.

Do at least four of the following problems.

1. A Toyota is sold for $8000.00 when new. Ten years later it is sold for $1000.00.

   a. What is the depreciation over the ten years?

   b. What percent of the new price is the depreciation?

2. A factory can produce 9600 bottles per day when it is operating at an 80% efficiency level. How many bottles are produced when it is at the following.

   a. a 100% efficiency level

   b. a 60% efficiency level

3. The lethal level of D.D.T. for a human is 500 parts per million.

   a. Express this as a decimal.

   b. Express this as a percent.

4. Express the following tax rates as mills.

   a. 2.57% b. 0.95%

   c. 11.5%

5. Express each of the following tax rates as a percent.

   a. 46.6 mills b. 2.59 mills

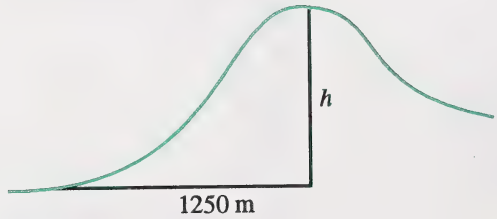
   c. 110.5 mills

6. How many mills are there in

   a. $1.00 b. $150.00

   c. 65¢

7. The slope of a hill is 7%. If the top of the hill is 1250 m away as shown in the diagram, how high is the hill?



For solutions to **Extensions**, turn to the **Appendix, Topic 2**.

Percents are often used to calculate interest or to determine rates of increase and decrease.

The following are a few examples.

Example 22

Find the simple interest on \$5000.00 for $3\frac{1}{2}$ years at 12%.

Solution:

$$\begin{aligned}\text{Interest} &= \text{principal} \times \text{rate} \times \text{time.} \\ &= P \times r \times t \\ &= 5000.00 \times 12\% \times 3\frac{1}{2} \\ &= 5000.00 \times 0.12 \times 3.5 \\ &= \$2100.00\end{aligned}$$

The interest on \$5000.00 for $3\frac{1}{2}$ years at 12% is \$2100.00. Here you used percent to find the interest on an investment.

Environment Canada, in their predictions, sometimes indicates the posssibility of precipitation as a percent.

Do you understand these statements?

- There is 100% chance of rain today.
- There is 10% chance of snow on the weekend.

Percent is also used in the Stock Market. Sometimes you hear that gold slipped so many percentage points or the dollar is down by $\frac{4}{100}$ of a cent. People who work in these areas make use of percents everyday.

Example 23

If your taxes go up $9\frac{1}{2}\%$, how much of an increase is there per one thousand dollars?

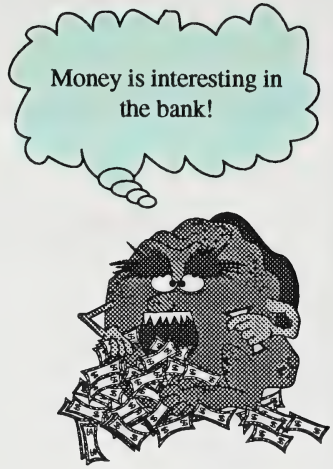
Solution:

Find $9\frac{1}{2}\%$ of \$1000.00.

This means **what number is $9\frac{1}{2}\%$ of \$1000.00?**

$$\begin{aligned}\frac{9.5}{100} &= \frac{x}{1000.00} \\ 100x &= 9.5 \times 1000.00 \\ \frac{100x}{100} &= \frac{9500.00}{100} \\ x &= 95.00\end{aligned}$$

You will be paying \$95.00 more on each \$1000.00 of taxes previously paid.



Now it is time for you to try some practical problems. Complete any three of the following problems.

8. a. What is the amount of interest if $3000.00 were invested for four years at 5% per year?

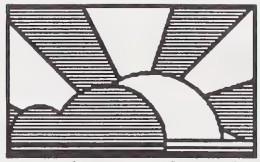
   b. How much money must be invested to yield $300.00 in interest at 5% per year over a period of four years?

9. What do you think 50% chance of rain means?

10. If your land tax was $550.00 last year and it went up 5%, how much will you pay for land tax this year?

11. Find 15% of $1500.00. Determine what 30% of $3000.00 will be.

For solutions to **Extensions**, turn to the **Appendix, Topic 2**.



Chance of rain may be 10%

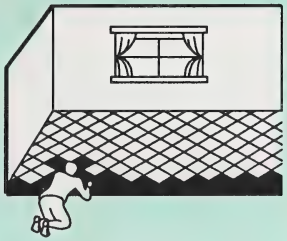
Chance of rain may be 60%.

Chance of rain is 100%.

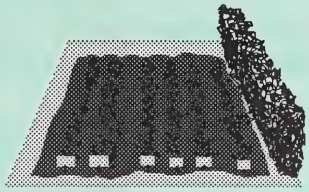
Chance of rain may be 90%.

Topic 3 Squares and Square Roots

Introduction



Finding the square of a number can help find the number of tiles needed to cover this floor.



Finding the square root of a number can help you find the length of the hedge. To do this you must know the area of the garden. The area of the garden pictured is 25 m^2.

Finding the square of a number and finding the square root of a number are inverse operations.

In this topic you will learn to square a number and how to find the square root of a number.

What Lies Ahead

Throughout the topic you will learn to

1. find the square of a number
2. find the square root of a number

Now that you know what to expect, turn the page to begin your study of squares and square roots.

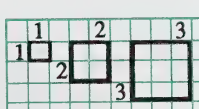
Exploring Topic 3

Activity 1

Find the square of a number.

What is the square of a number?

On the following grid, squares measuring 1 by 1, 2 by 2, and 3 by 3 have been drawn. In the same way, draw squares measuring 4 by 4, 5 by 5, and so on, all the way up to 10 by 10.



For each of the squares you have drawn, find the area by counting the grid squares it covers. Record your results in the following table.

Area of Squares:

Side length	Area
1	1
2	4
3	9
4	
5	

Side length	Area
6	
7	
8	
9	
10	

Can you tell what the area would be for a square having a side length of 11? Before you answer this question you may want to look at the numbers of the table that represent areas. There is a pattern. Do you agree?

$1 = 1 \times 1$ $\quad$ $36 = 6 \times 6$

$4 = 2 \times 2$ $\quad$ $49 = 7 \times 7$

$9 = 3 \times 3$ $\quad$ $64 = 8 \times 8$

$16 = 4 \times 4$ $\quad$ $81 = 9 \times 9$

$25 = 5 \times 5$ $\quad$ $100 = 10 \times 10$

You can now see that the area for a square having a side length of 11 is $11 \times 11 = 121$. You can say that 11 squared is 121 or that the square of 11 is 121.

More generally, the square of a number is a result of multiplying that number by itself. This holds for all rational numbers even if they are negative numbers. The squares for all rational numbers would be hard to show on the grid.

Example 1

Square the following.

- 14

  Solution:

  The square of 14 is $14 \times 14 = 196$.

- -4

  Solution:

  The square of -4 is $(-4) \times (-4) = 16$.

- 1.5

  Solution:

  The square of 1.5 is $1.5 \times 1.5 = 2.25$.

- 5.2

  Solution:

  The square of 5.2 is $5.2 \times 5.2 = 27.04$.

- $\frac{5}{6}$

Solution:

The square of $\frac{5}{6}$ is $\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$.

- $\frac{-3}{4}$

Solution:

The square of $\frac{-3}{4}$ is $\left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)=\frac{9}{16}$.

There is a special notation to indicate the square of a number. For example, the square of 2 can be written as 2^2 and the square of 11 can be written as 11^2. When using this notation to indicate the square of a **negative** number, you have to be careful. In this case the number to be squared must be put in parentheses.

$$(-5)^2=(-5)\times(-5)=25 \quad \text{whereas} \quad -5^2=-(5\times5)=-25$$

Example 2

Give the values for the following.

- 15^2

Solution:

$$15^2=15\times15=225$$

- $(-4)^2$

Solution:

$$(-4)^2=(-4)\times(-4)=16$$

- 2.5^2

Solution:

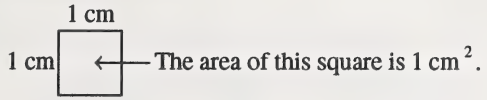
$$2.5^2=2.5\times2.5=6.25$$

- $\left(\frac{-2}{3}\right)^2$

Solution:

$$\left(\frac{-2}{3}\right)^2 = \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$$
$$= \frac{4}{9}$$

When squaring values in problems involving area, you should be aware of area units. The area of a square with a side length of 1 cm is one square centimetre. This quantity can be symbolized by using 1 cm^2.



Similarly 1 m^2 is the area of a square measuring 1 m by 1 m. You can think of m^2 to be the square of a metre and cm^2 to be the square of cm.

Example 3

Find the area of a square having a side length of 15 cm.

Solution:

$$A = s^2$$
$$= (15 \text{ cm})^2$$
$$= 15 \text{ cm} \times 15 \text{ cm}$$
$$= 225 \text{ cm}^2$$

The area of the square is 225 cm^2.

Example 4

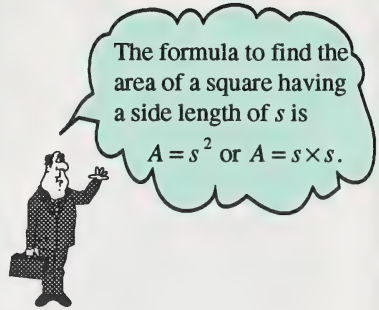
Find the area of a square floor whose walls are 3.6 m long.

Solution:

$$A = s^2$$
$$= (3.6 \text{ m})^2$$
$$= 3.6 \text{ m} \times 3.6 \text{ m}$$
$$= 12.96 \text{ m}^2$$

The area of the floor is 12.96 m^2.

You can use the multiplication key on a calculator to multiply a number by itself. You can also find the square more directly by using the x^2 key.



Example 5

Square the following using your x^2 key. Look at the text following Example 5 for assistance as to what is to be done if your calculator does not have this key.

- Square 25.

  Solution:

Enter	Display
25	25
x^2	625

- Square 14.7.

  Solution:

Enter	Display
14.7	14.7
x^2	216.09

On some calculators you may have to punch the Inv key and the $\sqrt{\ }$ key to get x^2.

If your calculator does not have a x^2 key, you may square 25 by using your calculator like this.

Enter	Display
25	25
×	25
25	25
=	625

or

Enter	Display
25	25
×	25
=	625

Do any eight of the following.

1. A large square has been drawn on grid paper. If the square has a side of 36 grid units, how many grid squares would it cover?

2. What is 16 squared?

3. Calculate $(-21)^2$.

Use your calculator for questions 4, 5, and 6.

4. Give the square of 3.15.

5. Calculate 38^2.

6. Calculate $(3.5)^2$.

7. To cover the square floor of a study, Mr. Abdul needs to lay 11 square linoleum tiles along each wall. How many tiles will he need to cover the whole floor?

8. Noel wants to finish redecorating her bathroom. She has one square wall left to do. The height of the wall is 2.47 m. The wallpaper she has left will cover 6 m^2. Will Noel have enough wallpaper to finish the project?

9. Find the area in cm^2 of a square having a side length of 8 cm.

10. The formula for the area of a circle is $A = \pi r^2$. Suppose the radius of a circle is 15 cm. What is the area of the circle to the nearest square centimetre?

For solutions to **Activity 1**, turn to the **Appendix, Topic 3**.

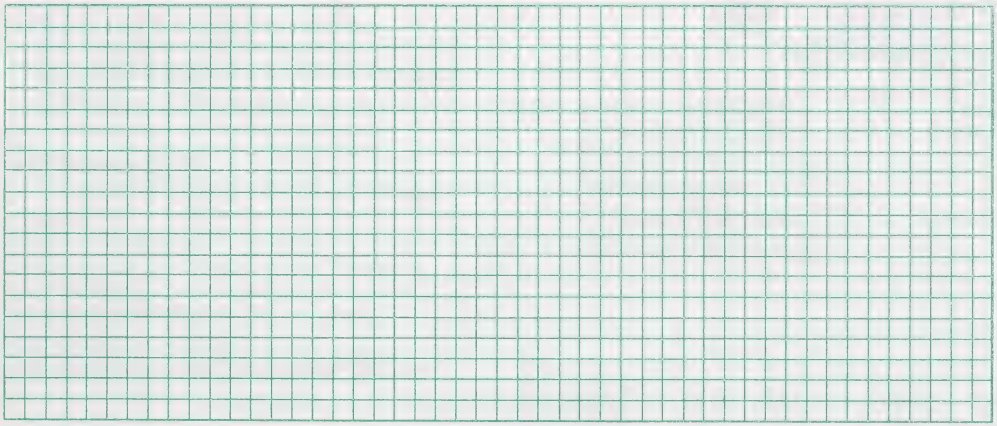
Drawing diagrams usually helps in solving word problems.

In the formula $A = \pi r^2$, π is a constant which is equal to approximately 3.14 when rounded to two decimal places. It can also be $\frac{22}{7}$. r represents the radius of a circle.

Activity 2

Find the square root of a number.

On the following grid, draw a square so that it covers 49 grid squares. What is the side length for the square you drew?



It may have taken you a few tries but you should find that the square you have drawn has a side length of 7 units. If the square you drew was to have an area of 64, then its side length would be 8. It is said that 7 is a square root of 49 and 8 is a square root of 64. Finding a square root is the opposite operation to that of finding the square of a number.

9 squared is the same as 9^2.

$$\begin{aligned} 9 \text{ squared} &= 9^2 \\ &= 9 \times 9 \\ &= 81 \end{aligned}$$

The square root of 81 is the same as $\sqrt{81}$.

$$\begin{aligned} \text{The square root of } 81 &= \sqrt{81} \\ &= 9 \end{aligned}$$

If you write a number as the product of two equal factors, the factor is a square root of the number. It follows from $4 = 2 \times 2$ that 2 is a square root of 4. Also, since $9 = 3 \times 3$, 3 is a square root of 9. Instead of writing out **the square root of**, the symbol $\sqrt{\ }$ can be used.

For example, $\sqrt{9} = 3$ and $\sqrt{49} = 7$.

Take another look at the square roots of 4, 9, 49, and 64. For each of these there is a positive number that is a square root. Are there any other possibilities?

These numbers can also be factored as follows.

$4 = (-2) \times (-2)$

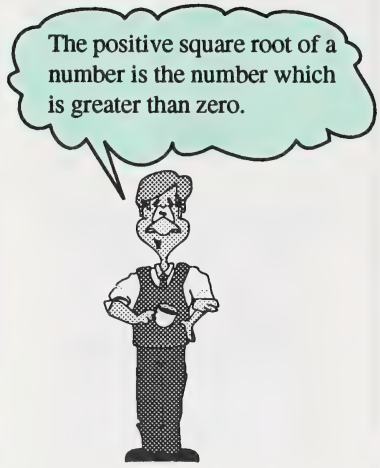
$9 = (-3) \times (-3)$

$49 = (-7) \times (-7)$

$64 = (-8) \times (-8)$

This means that -2 is a square root of 4, -3 is a square root of 9, -7 is a square root of 49, and -8 is a square root of 64. Although each number has two square roots, one positive and one negative, the term **square root** usually refers to the positive square root. Generally, the symbol is used to refer to the positive square root. Thus, $\sqrt{64} = +8$ or 8 and $\sqrt{81} = +9$ or 9.

In $25^2 = 625$ and $\sqrt{625} = 25$, 25^2 and $\sqrt{625}$ are opposite operations. Opposite operations are sometimes called inverse operations.



A **perfect square** is a number that has a whole number as its square root. From the following chart, select the perfect squares between 1 and 100.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	21	4.583	41	6.403	61	7.810	81	9.000
2	1.414	22	4.690	42	6.481	62	7.784	82	9.055
3	1.732	23	4.796	43	6.557	63	7.937	83	9.110
4	2.000	24	4.899	44	6.633	64	8.000	84	9.165
5	2.236	25	5.000	45	6.708	65	8.062	85	9.220
6	2.449	26	5.099	46	6.782	66	8.124	86	9.274
7	2.646	27	5.196	47	6.856	67	8.185	87	9.327
8	2.828	28	5.292	48	6.928	68	8.246	88	9.381
9	3.000	29	5.385	49	7.000	69	8.307	89	9.434
10	3.062	30	5.477	50	7.071	70	8.367	90	9.487
11	3.317	31	5.568	51	7.141	71	8.426	91	9.539
12	3.464	32	5.657	52	7.211	72	8.485	92	9.592
13	3.606	33	5.745	53	7.280	73	8.544	93	9.644
14	3.742	34	5.831	54	7.348	74	8.602	94	9.695
15	3.873	35	5.916	55	7.416	75	8.660	95	9.747
16	4.000	36	6.000	56	7.483	76	8.718	96	9.798
17	4.123	37	6.083	57	7.550	77	8.775	97	9.849
18	4.243	38	6.164	58	7.616	78	8.832	98	9.899
19	4.359	39	6.245	59	7.681	79	8.888	99	9.950
20	4.472	40	6.325	60	7.746	80	8.944	100	10.000

The perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

Each member of the previous list is the square of the first ten natural numbers.

Using the chart, look at the square root of 53. It is given as 7.280. However, $7.280 \times 7.280 = 52.9984$. This shows that many of the square roots listed are approximations. They are rounded to the nearest thousandth. A chart such as the one provided here can be used to give adequate approximations of square roots.

Example 6

A square garden has an area of 75 m^2. What is the length of each side? Round the length to the nearest tenth.

Solution:

From the table $\sqrt{75} = 8.660 \doteq 8.7$ (to the nearer tenth).

The length of each side is approximately 8.7 m.

Sometimes you can mentally make rough approximations of square roots by using perfect squares.

Example 7

Find the square root of 54 to the nearest tenth without using the table.

Solution:

$\sqrt{49} < \sqrt{54} < \sqrt{64}$

$7 < \sqrt{54} < 8$

54 is about $\frac{1}{3}$ of the way from 49 to 64.

Since the $\sqrt{54}$ is about $\frac{1}{3}$ of the way from 7 to 8, the $\sqrt{54}$ is approximately 7.3.

The $\sqrt{\ }$ button on your calculator allows you to determine square roots easily.

Example 8

Determine $\sqrt{1369}$.

Solution:

Enter	Display
1369	1369
$\sqrt{\ }$	37

Use your calculator to check some of the values in the chart. Do the calculator and chart agree? They should if you round your calculator answer to three decimal places. The calculator is very useful because it gives square roots for numbers not found in the chart and it displays values with greater accuracy.

Notice how the value of $\frac{1}{3}$ is found using 49, 54, and 64.

$54 - 49 = 5$

$64 - 49 = 15$

Use 5 and 15 to get $\frac{5}{15}$ or $\frac{1}{3}$. Thus, the square root of 54 becomes $\sqrt{54} \doteq 7 + \frac{1}{3}$.

$\doteq 7.3$

The symbol < means **is less than** and > means **is greater than**. In the estimation of the $\sqrt{54}$ did you notice that the 49 and 64 in $\sqrt{49} < \sqrt{54} < \sqrt{64}$ are both perfect squares?

Example 9

Find $\sqrt{75}$ to the nearest ten thousandth.

Solution:

Enter	Display
75	75
$\sqrt{}$	8.660254

To the nearest ten thousandth,
$\sqrt{75} \doteq 8.6603$.

When applying the rules of order of operations, consider the square root sign as a bracket. That is, do the operations under the root sign first.

Example 10

Evaluate $\sqrt{25-16}+6\times 8$.

Solution:

$$\begin{aligned}\sqrt{25-16}+6\times 8 &= \sqrt{9}+6\times 8\\ &= 3+48\\ &= 51\end{aligned}$$

1. Do all of the following questions.

   a. $\sqrt{121}$ b. $\sqrt{81}$

   c. $\sqrt{49}$ d. $\sqrt{196}$

   e. $\sqrt{6889}$ f. $\sqrt{5^2-3^2}$

2. Do all of the following questions. Give your answers rounded to three decimal places.

   a. $\sqrt{10}$ b. $\sqrt{40}$

   c. $\sqrt{56}$ d. $\sqrt{255}$

   e. $\sqrt{6401}$

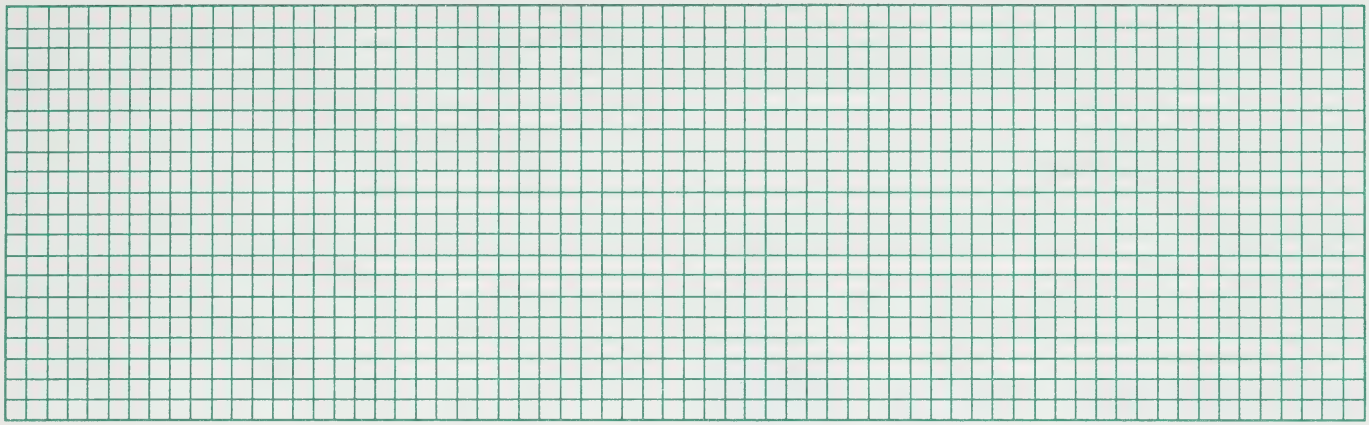
Use this memory device when several operations in an expression are to be simplified.

BEDMAS

1. Brackets
2. Exponents
3. Division
4. Multiplication
5. Addition
6. Subtraction

Complete all of the remaining problems.

3. What is the length of a side of a square having an area of 121 grid squares? You may use the grid to work out your answer.



4. What length of fence would you need to enclose a square field which has an area of 1155 m^2? Give your answer to the nearest metre.

5. The area of a circle is 314 cm^2. Find the radius by using the formula $A = \pi r^2$. (Use $\pi = 3.14$.)

6. Estimate the square root of each of the following to one decimal place without using a calculator or a chart.

   a. $\sqrt{11}$ b. $\sqrt{38}$ c. $\sqrt{95}$

For solutions to **Activity 2**, turn to the **Appendix, Topic 3**.

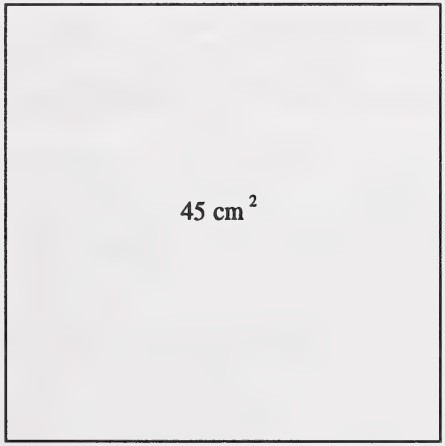
If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

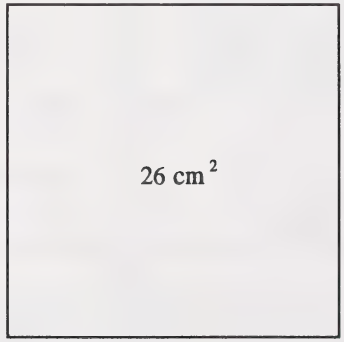
Some square roots are not integers. You may have trouble applying these to anything real. The following square has an area of 45 cm^2.



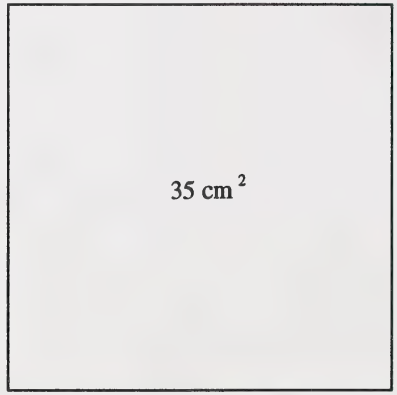
The length of a side represents the square root of 45. When measured with a ruler, the side length is 6.7 cm rounded to one decimal place. Therefore, the square root of 45 is 6.7 cm rounded to one decimal place. In the questions that follow, you can measure side lengths in order to arrive at the square roots.

Determine each of the following by measuring the side length of the appropriate square. Give your answer to one decimal place.

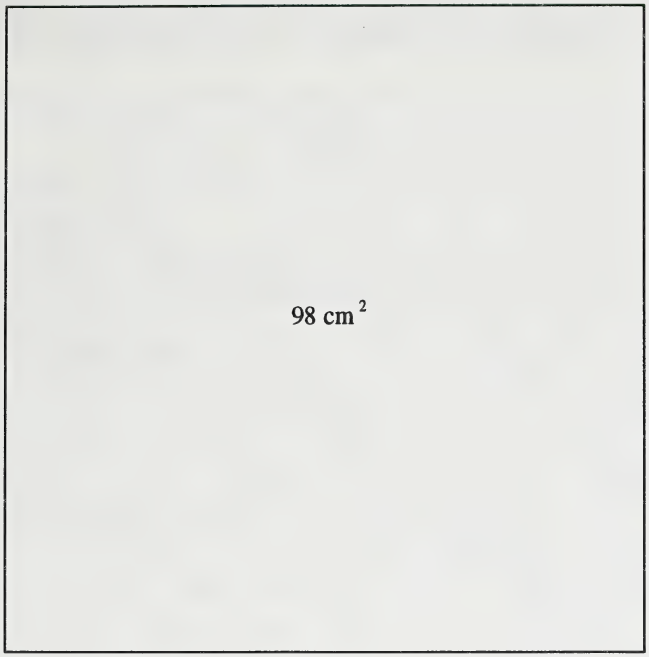
1. $\sqrt{26}$



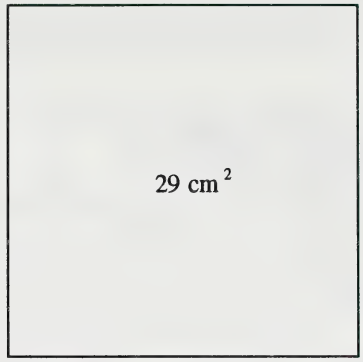
2. $\sqrt{35}$



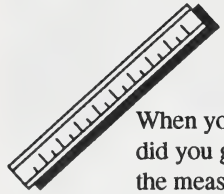
3. $\sqrt{98}$



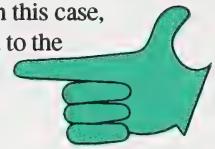
4. $\sqrt{29}$



For solutions to **Extra Help**, turn to the **Appendix**, **Topic 3**.



When you measured the length of the sides did you get an **exact** measurement? No, all the measurements are rounded. In this case, your measurements were rounded to the nearest tenth of a centimetre.



Extensions

There is a simple way to mentally calculate the squares of numbers ending in 5. The method can be shown by finding the square of 45.

The square of **45** will end with the square of the **5**. The square ends in 25. The other digits are found by multiplying the 4 by the next larger digit which is 5.

$$45^2 = 2025$$
$$4 \times 5 = 20$$

Use this shortcut to find 25^2.

$$25^2 = 625$$
$$2 \times 3 = 6$$

The 2 is followed by 3.

This method also works for three digit numbers. When this is the case it becomes harder to perform the calculation in your head. Take for example 105^2.

$$105^2 = 11025$$
$$10 \times 11$$

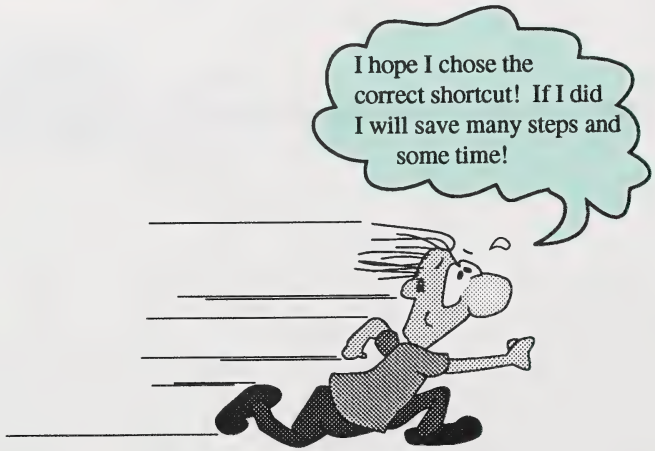
The 10 is followed by 11.

In the following questions, you can apply the previous method.

Mentally calculate the following.

1. 15^2
2. 35^2
3. 55^2
4. 65^2
5. 85^2
6. 95^2
7. 1005^2

For solutions to **Extensions**, turn to the **Appendix**, **Topic 3**.



Topic 4 Laws of Exponents

Introduction

Bacteria reproduces by splitting into two. In conditions that favor them, bacteria can mature in twenty minutes after a split. They then can split again and again. At this rate, a single bacterium becomes two in twenty minutes. In another twenty minutes, there would be four. In a further twenty minutes, there would be eight and so on.

Continuing in this way, there would be over two million bacteria in just seven hours. There are 21 twenty-minute periods in seven hours. The bacteria population at the end of seven hours can be found by evaluating the power 2^{21}. This value turns out to be 2 097 152.

In this topic, you will learn more about powers.

What Lies Ahead

Throughout this topic you will learn to

1. evaluate powers such as 5^6, 94^2, $(-11)^3$, and $(-3)^5$
2. use the Product Law to simplify expressions such as $(4^8)(4^3)$ and $(5^4)(5^{13})$
3. use the Quotient Law to simplify expressions such as $\frac{7^5}{7^3}$ and $\frac{2^{12}}{2^{11}}$
4. evaluate powers with zero and negative exponents such as 4^0, 7^0, and 6^{-1}

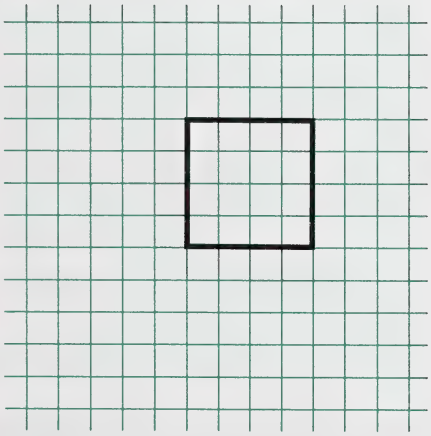
Now that you know what to expect, turn the page to begin your study of laws of exponents.

Exploring Topic 4

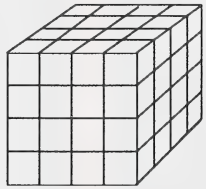
Activity 1

Evaluate powers such as 5^6, 94^2, $(-11)^3$, and $(-3)^5$.

You now know what the square of a number is and how it can be represented on a grid. For example $4^2 = 4 \times 4$ and is represented by the area of a square which has a side length of 4. This is shown as follows.



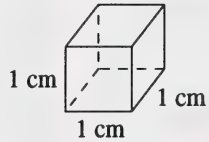
A number cubed may be represented by the volume of a cube. For example, 4 cubed is represented by the number of unit cubes in the larger cube with a side length of 4.



Each layer can be seen to contain 4×4 unit cubes. All four layers together contain $4 \times 4 \times 4$ unit cubes. You can say that 4 cubed equals $4 \times 4 \times 4$ or 64. This can also be expressed as 4^3. Thus, $4^3 = 4 \times 4 \times 4$

$= 64$

When cubing values in problems involving volume, you will need volume units to express your answer. The volume of a cube with a side length of 1 cm is one cubic centimeter. This is symbolized as $1\ cm^3$.



In the same way $1\ m^3$ is the volume of a cube measuring 1 m by 1 m by 1 m. Volume units can be seen to be the result of cubing given units of length.

A unit cube is a cube which measures 1 cm by 1 cm by 1 cm. It occupies a space which is 1 cubic cm or $1\ cm^3$. The space it occupies is called volume.

Example 1

Find the volume of a cube having a side length of 4 m.

Solution:

$$v = s^3$$
$$= (4\text{ m})^3$$
$$= 4\text{ m} \times 4\text{ m} \times 4\text{ m}$$
$$= 64\text{ m}^3$$

The volume is 64 m^3.

In the expression 4^3,

4^3 is the power,
4 is the base,
3 is the exponent, and
$4 \times 4 \times 4$ is the expanded form.

In a power, the base and exponent can take on many values. The exponent tells how many times the base is to be used as a factor in the expanded form or in the multiplication problem.

Example 2

Express the following in expanded form.

- 7^5

  Solution:

  $7^5 = 7 \times 7 \times 7 \times 7 \times 7$

- 14^4

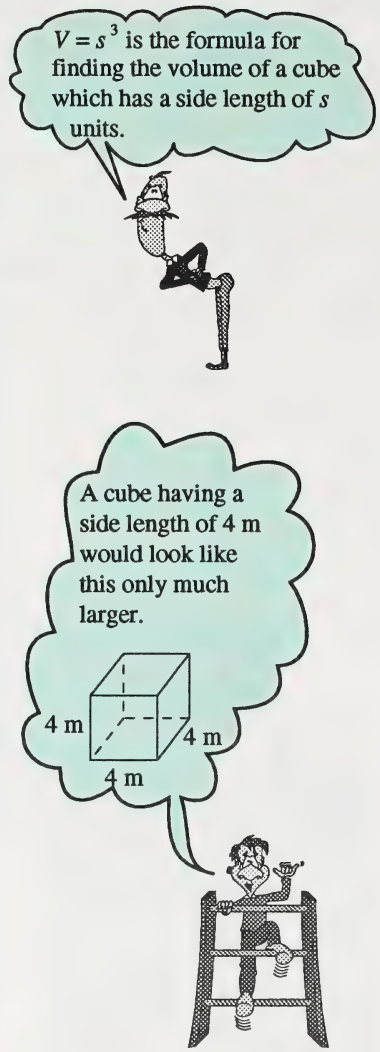
  Solution:

  $14^4 = 14 \times 14 \times 14 \times 14$

- 12^6

  Solution:

  $12^6 = 12 \times 12 \times 12 \times 12 \times 12 \times 12$


$V = s^3$ is the formula for finding the volume of a cube which has a side length of s units.
A cube having a side length of 4 m would look like this only much larger.
4 m
4 m
4 m

The base does not have to be a whole number. It can be any integer.

Example 3

Express the following in expanded form.

- $(-12)^3$

  Solution:

  $(-12)^3 = (-12)\times(-12)\times(-12)$

- $(-25)^5$

  Solution:

  $(-25)^5 = (-25)\times(-25)\times(-25)\times(-25)\times(-25)$

When the base is negative, it is enclosed in a parenthesis. This is to make clear what the base really is. Study the following and notice the difference.

$-2^4 = -(2\times2\times2\times2) = -16$ whereas $(-2)^4 = (-2)\times(-2)\times(-2)\times(-2) = +16$

Can you see the difference between -2^4 and $(-2)^4$?

Notice how these common number systems are different and how they are similar:

$N = 1, 2, 3, \ldots$

$W = 0, 1, 2, 3, \ldots$

$I = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Example 4

Evaluate the following.

- 3^5

  Solution:

  $$\begin{aligned} 3^5 &= 3\times3\times3\times3\times3 \\ &= 243 \end{aligned}$$

- 5^4

  Solution:

  $$\begin{aligned} 5^4 &= 5\times5\times5\times5 \\ &= 625 \end{aligned}$$

- $(-4)^3$

  Solution:

  $$\begin{aligned} (-4)^3 &= (-4)\times(-4)\times(-4) \\ &= -64 \end{aligned}$$

- -4^3

  Solution:

  $$\begin{aligned} -4^3 &= -(4\times4\times4) \\ &= -64 \end{aligned}$$

  Note that in -4^3, the base is 4 and not -4.
  Thus, the base in -4^3 is positive.

- -2^1

  Solution:

  $$-2^1 = -2$$

Notice how powers are read.

- Read 3^5 as 3 to the fifth power or 3 to the power of 5.
- Read 5^4 as 5 to the fourth power or 5 to the power of 4.

In evaluating powers, how can you tell when your answers will be negative? Look at the following examples.

$$(-5)^2 = (-5)\times(-5)$$
$$= 25$$

$$5^2 = 5\times 5$$
$$= 25$$

$$(-5)^3 = (-5)\times(-5)\times(-5)$$
$$= -125$$

$$5^3 = 5\times 5\times 5$$
$$= 125$$

$$(-3)^4 = (-3)\times(-3)\times(-3)\times(-3)$$
$$= 81$$

$$(-3)^5 = (-3)\times(-3)\times(-3)\times(-3)\times(-3)$$
$$= -243$$

Did you make these observations?

- Unless the base is negative, the answer is always positive.
- If the base is negative, the exponent must be **odd** for the answer to be negative.

Example 5

For each of the following, tell whether the result will be positive or negative.

- 4^{12}

  Solution:

  4^{12} is positive since the base is positive.

- $(-5)^7$

  Solution:

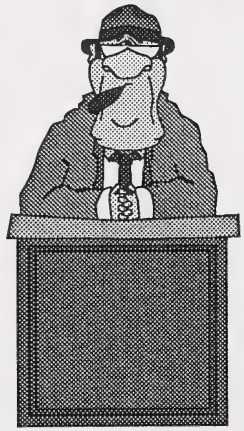
  $(-5)^7$ is negative since the base is negative and the exponent is odd.

- $(-41)^5$

  Solution:

  $(-41)^5$ is negative since the base is negative and the exponent is odd.

For the value of a power to be negative, **both** of the following conditions must exist.
- The base is negative.
- The exponent is odd.



- -7^{12}

Solution:

-7^{12} is negative since 7^{12} itself is positive but the negative sign in front of the power makes the value negative.

- 14^{5}

Solution:

14^{5} is positive since the base is positive.

It can be difficult to evaluate a power by putting it into expanded form and multiplying through. Imagine evaluating 2^{30}. That would take some doing. In such cases, the calculator becomes very useful. The y^x key allows you to evaluate powers. Not all calculators are the same. Some calculators have a x^y key instead of y^x. On others the Inv key must be used. Refer to your manual for the proper use of your particular calculator.

Example 6

- Evaluate 5^2 using your calculator.

Solution:

Enter	Display	or	Enter	Display
5	5		5	5
y^x or x^y	5		x^2	25
2	2			
=	25			

- Evaluate $(-7)^3$ using your calculator.

Solution:

Enter	Display
7	7
+/-	−7
y^x or x^y	−7
3	3
=	−343

- Evaluate 3^{10} using your calculator.

  Solution:

Enter	Display
3	3
y^x or x^y	3
10	10
=	59049

Now do some practice on exponents.

1. Express each as a single power.

   a. $23\times23\times23\times23$

   b. $(-6)\times(-6)\times(-6)\times(-6)$

   c. $8\times8\times8\times8\times8\times8$

2. Evaluate at least three of the following.

   a. 5^4 b. 7^2

   c. 12^3 d. 2^6

   e. 3^4

3. Evaluate at least three of the following.

   a. $(-4)^3$ b. -2^4

   c. $-(-3)^5$ d. 8^3

   e. $(-10)^6$

4. Evaluate at least three of the following.

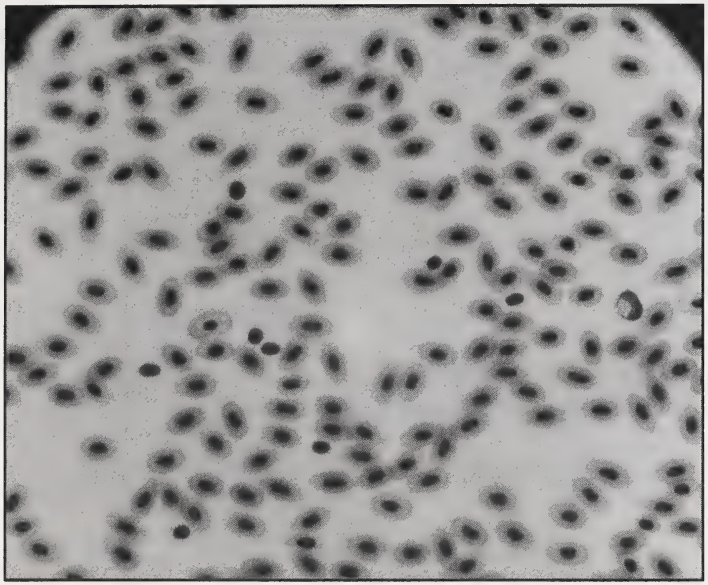
   a. 24^5 b. 64^4

   c. 8^8 d. 2^{21}

   e. 549^1

5. Under favorable conditions, a population of fruit flies can quadruple every five days. (Quadruple means four times the amount.) At this rate, the fruit fly population could increase by a factor of 4^6 over the course of 30 days. Suppose a laboratory started with ten fruit flies and they reproduced at the rate described. How many fruit flies would the lab have in thirty days? (Hint: The total number of fruit flies is equal to the number of fruit flies you started with times the factor of 4^6.)

6. The population of a certain bacteria doubles every twenty minutes. If there are ten bacteria in a test tube at the start, how many bacteria will there be in five hours? (Note: There are three twenty-minute periods in one hour. Since the bacteria are doubling, the base is two. In one hour the rate of increase will be 2^3. In five hours the rate of increase is $2^{3\times5}$ or 2^{15}. The total number of bacteria is equal to the number you started with times the rate of increase.)



For solutions to **Activity 1**, turn to the **Appendix, Topic 4.**

Activity 2

Use the Product Law to simplify expressions such as $(4^8)(4^3)$ and $(5^4)(5^{13})$.

The Product Law can make it easier to deal with products of powers. Consider $3^2 \times 3^4$. When each of the factors is expressed in expanded form, the product becomes $\underbrace{3\times3}_{2\text{ factors}} \times \underbrace{3\times3\times3\times3}_{4\text{ factors}}$. There are six factors of 3. Expressed as a single power notation this is 3^6.

This shows that $3^2 \times 3^4 = 3^6$ where the exponent 6 in the product is the sum of 2 and 4.

Look at a few more of such products.

- $5^4 \times 5^7 = \underbrace{5\times5\times5\times5}_{4\text{ factors}} \times \underbrace{5\times5\times5\times5\times5\times5\times5}_{7\text{ factors}}$

  $= 5^{4+7}$

  $= 5^{11}$

- $(15)^8 \times (15)^5 = \underbrace{15\times15\times15\times15\times15\times15\times15\times15}_{8 \text{ factors}} \times \underbrace{15\times15\times15\times15\times15}_{5 \text{ factors}}$

$= 15^{8+5}$

$= 15^{13}$

Have you discovered any pattern? When **multiplying** two powers with **the same** base, the exponents are **added** and the common base is used. What happens when the bases are not the same?

Examine $2^3 \times 5^2 = 2\times2\times2 \times 5\times5$.

When the bases are **different**, the product **cannot** be found by simply adding the exponents and repeating the base.

Example 7

Using your calculator, show that $3^8 \times 3^4 = 3^{12}$.

Solution:

3^8

Enter	Display
3	3
x^y or y^x	3
8	8
=	6561

3^4

Enter	Display
3	3
x^y or y^x	3
4	4
=	81

3^{12}

Enter	Display
3	3
x^y or y^x	3
12	12
=	531441

Check:

LS	RS
$3^8 \times 3^4$	3^{12}
6561×81	531 441
531 441	531 441

LS = RS

The previous example is another way of showing that exponents are added in multiplication of powers having the same base.

Example 8

Express $5^4 \times 5$ as a single power.

Solution:

$$\underbrace{5 \times 5 \times 5 \times 5}_{5^4} \times \underbrace{5}_{5^1} = 5^5$$

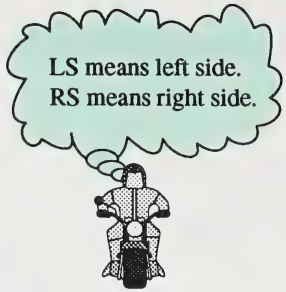
If exponents were to be added to give the answer in the previous example, you would have to consider 5 to have the exponent 1. You can say the following:

- $45 = 45^1$
- $(-5) = (-5)^1$
- $25 = 25^1$

Any number can be considered to be a power with exponent 1.

When multiplying powers with the same base, the exponents are added.

The Product Law states that $x^a \times x^b = x^{a+b}$.



Example 9

- Express $5^3 \times 5^{14}$ as a single power.

  Solution:

$$5^3 \times 5^{14} = 5^{3+14}$$
$$= 5^{17}$$

- Express $(-4)^{21} \times (-4)^4$ as a single power.

  Solution:

$$(-4)^{21} \times (-4)^4 = (-4)^{21+4}$$
$$= (-4)^{25}$$

- Express $10^5 \times 10$ as a single power.

  Solution:

$$10^5 \times 10 = 10^5 \times 10^1$$
$$= 10^{5+1}$$
$$= 10^6$$

- Express $3^{41} \times 3^{18}$ as a single power.

  Solution:

$$3^{41} \times 3^{18} = 3^{41+18}$$
$$= 3^{59}$$

Now use this rule to do the following questions.

Express each of the following as a single power.

1. $9^{10} \times 9^{20}$
2. $(-4) \times (-4)^{20}$
3. $41^{17} \times 41^{26}$
4. $120^2 \times 120^{30}$
5. $(-32)^7 \times (-32)^7$
6. $21^{16} \times 21^2$

For solutions to **Activity 2**, turn to the **Appendix, Topic 4**.

Activity 3

Use the Quotient Law to simplify expressions such as $\frac{7^5}{7^3}$ and $\frac{2^{12}}{2^{11}}$.

You have learned how to multiply when working with powers with the same base, but what happens if you are dividing powers with the same base?

Look at $\frac{5^7}{5^3}$. This equals $\frac{5\times5\times5\times5\times5\times5\times5}{5\times5\times5}$.

This can be simplified as follows.

$$\frac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times5\times5\times5\times5}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}} = 5\times5\times5\times5$$

$$= 5^4$$

From this you can see that $\frac{5^7}{5^3} = 5^4$.

Consider another quotient being simplified by putting the numerator and denominator in expanded form and then dividing through by the common factors.

$$25^{11} \div 25^9 = \frac{25^{11}}{25^9}$$

$$= \frac{\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times\overset{1}{\cancel{25}}\times25\times25}{\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}\times\underset{1}{\cancel{25}}}$$

$$= 25\times25$$

$$= 25^2$$

The following is simplifying by the elimination of common factors.

$$\frac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times5\times5\times5\times5}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}} = 5^4$$

Did you see any shortcuts in finding the answer? It seems that the exponents are **subtracted** when the **bases** are the **same**. What happens when the bases are not the same?

$$3^5 \div 4^3 = \frac{3^5}{4^3}$$
$$= \frac{3\times3\times3\times3\times3}{4\times4\times4}$$

Putting the numerator and denominator in expanded form does not show common factors. When the bases are **different** the quotient **cannot** be simplified by subtracting exponents. You must divide to find the answer.

$$3^5 \div 4^3 = 243 \div 64$$
$$= 3.797 \quad \text{(to three decimal places)}$$

Example 10

Using your calculator, show that $7^8 \div 7^6 = 7^2$.

Solution:

7^8

Enter	Display
7	7
x^y or y^x	7
8	8
=	5764801

7^6

Enter	Display
7	7
x^y or y^x	7
6	6
=	117649

7^2

Enter	Display
7	7
x^y or y^x	7
2	2
=	49

Check

LS	RS
$7^8 \div 7^6$	7^2
5 764 801 ÷ 117 649	49
49	49
LS =	RS

This example is another way of showing that exponents are subtracted in the division of powers with the same base.

Example 11

Express $6^4 \div 6$ as a single power.

Solution:

$$6^4 \div 6 = \frac{6 \times 6 \times 6 \times 6}{6}$$
$$= 6 \times 6 \times 6$$
$$= 6^3$$

If exponents were subtracted to find the answer in the previous example, you would have to consider the 6 to have an exponent of 1. That is, $6 = 6^1$. As you saw earlier with multiplication, any number can be considered to be a power with an exponent of 1.

When dividing powers with the same base, the exponents are subtracted.

The Quotient Law states that

$\frac{x^a}{x^b} = x^{a-b}$ or $x^a \div x^b = x^{a-b}$.

Example 12

- Express $5^3 \div 5^2$ as a single power.

  Solution:

  $$5^3 \div 5^2 = 5^{3-2}$$
  $$= 5^1 \text{ or } 5$$

- Express $\frac{15^{23}}{15^{20}}$ as a single power.

  Solution:

  $$\frac{15^{23}}{15^{20}} = 15^{23-20}$$
  $$= 15^3$$

- Express $8^{41} \div 8^{39}$ as a single power.

  Solution:

  $$8^{41} \div 8^{39} = 8^{41-39}$$
  $$= 8^2$$

When any number is written without an exponent, it is understood to have an exponent of one as in $6 = 6^1$ and $92 = 92^1$.

In the Quotient Law, x cannot equal zero because

$$\frac{x^a}{x^b} = \frac{0^a}{0^b}$$
$$= \frac{0}{0}$$

Dividing by 0 is not possible.

- Express $\frac{(-3)^{10}}{(-3)^{9}}$ as a single power.

Solution:

$$\frac{(-3)^{10}}{(-3)^{9}} = (-3)^{10-9}$$
$$= (-3)^{1} \text{ or } -3$$

Example 13

- Evaluate $7^{21} \div 7^{19}$.

Solution:

$$7^{21} \div 7^{19} = 7^{2}$$
$$= 49$$

- Evaluate $28^{16} \div 28^{13}$.

Solution:

$$28^{16} \div 28^{13} = 28^{3}$$
$$= 21\,952$$

See if you can apply this rule to the questions which follow.

1. Express each of the following as a single power.

   a. $9^{10} \div 9^{9}$

   b. $10^{27} \div 10^{4}$

   c. $14^{12} \div 14$

   d. $\frac{272^{12}}{272^{11}}$

   e. $\frac{3^{60}}{3^{30}}$

   f. $11^{15} \div 11^{5}$

2. Evaluate the following.

   a. $314^{7} \div 314^{6}$

   b. $606^{112} \div 606^{111}$

   c. $8^{43} \div 8^{40}$

For solutions to **Activity 3**, turn to the **Appendix**, **Topic 4**.

Activity 4

Evaluate powers with zero and negative exponents such as 4^0, 7^0, and 6^{-1}.

So far, you can think of the exponent as telling how many factors there are in the expanded form. For example $5^3 = 5\times5\times5$. What about zero as an exponent or negative exponents? What does 4^0 mean and what does 4^{-2} mean?

Maybe you would like to see what your calculator does with a power which has zero as an exponent.

Example 14

Using your calculator, evaluate 4^0 and 10^0.

Solution:

4^0

Enter	Display
4	4
x^y or y^x	4
0	0
=	1

Thus, $4^0 = 1$.

10^0

Enter	Display
10	10
x^y or y^x	10
0	0
=	1

Thus, $10^0 = 1$.

There is another way to evaluate powers with zero exponents.

Using the Quotient Law to express $3^4 \div 3^4$ as a single power, you get $3^{4-4} = 3^0$. (**Remember:** $\frac{x^a}{x^b} = x^{a-b}$)

$3^4 \div 3^4 = 3^0$ (Quotient Law)

$3^4 \div 3^4 = \frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}}$ (elimination of common factors)

$= 1$

Since $3^4 \div 3^4 = 3^0$ and $3^4 \div 3^4 = 1$, then $3^0 = 1$.
Following is another example.

$2^5 \div 2^5 = 2^{5-5}$ (Quotient Law)

$= 2^0$

$2^5 \div 2^5 = \frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}} = 1$ (elimination of common factors)

Since $2^5 + 2^5 = 2^0$ and $2^5 + 2^5 = 1$, then $2^0 = 1$.

You can also consider multiplication to get a value for a power with a zero exponent.

Consider $5^4 \times 5^0$. The Product Law tells you that $5^4 \times 5^0$, expressed as a single power, becomes $5^{4+0} = 5^4$. That is, $5^4 \times 5^0 = 5^4$. For this to be true, 5^0 would have to equal 1.

Use your calculator to try some other powers with exponents of zero? You should get the same answer each time, even when you use negative bases. The answer is always 1.

Think of how you would finish the following sentence.

A power with an exponent of zero _____

_____.

You should have finished the sentence with **is equal to 1**. Note, however, that there is a restriction. The base cannot be zero. If you try to evaluate 0^0 on your calculator, you will get an error message. You cannot use the quotient rule to help either because, that would involve a zero in the denominator of a fraction.

You can now conclude that a power with a nonzero base and an exponent of zero is equal to 1.

The zero exponent in a power means that $x^0 = 1$, where $x \neq 0$.

Now, what about negative exponents? How can they describe the number of factors in the expanded form? Maybe the quotient rule can help again in evaluating powers with negative exponents.

Consider the following:

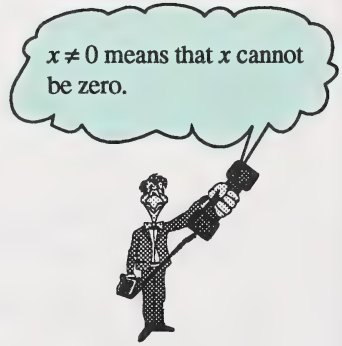
$$2^3 \div 2^5 = 2^{3-5}$$
$$= 2^{-2}$$

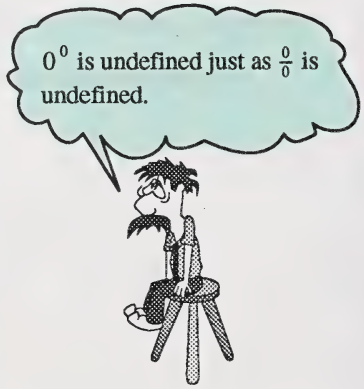
Evaluate the previous equation using expanded form.

$$2^3 \div 2^5 = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times 2 \times 2}$$
$$= \frac{1}{2 \times 2}$$
$$= \frac{1}{2^2}$$

From this you can see that $2^{-2} = \frac{1}{2^2}$.

$x^a \times x^b = x^{a+b}$





Consider another example.

$5^2 + 5^4 = 5^{2-4}$

$= 5^{-2}$

Also, $5^2 + 5^4 = \dfrac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}} \times 5 \times 5}$

$= \dfrac{1}{5 \times 5}$

$= \dfrac{1}{5^2}$

Therefore, $5^{-2} = \frac{1}{5^2}$.

This leads to the conclusion that a power with a negative exponent is equal to the reciprocal of the corresponding power with the exponent made positive. Again, the restriction that the base cannot be zero applies.

A negative exponent means that $x^{-a} = \frac{1}{x^a}$ where $x \neq 0$.

The calculator can be used to check this relationship. For this you will need the reciprocal key [1/x]. The [1/x] key is usually found as a 2nd function key. Press the [2nd F] or [Inv] key immediately before pressing the [1/x] key.

Example 15

Use your calculator to show that $2^{-3} = \frac{1}{2^3}$.

Solution:

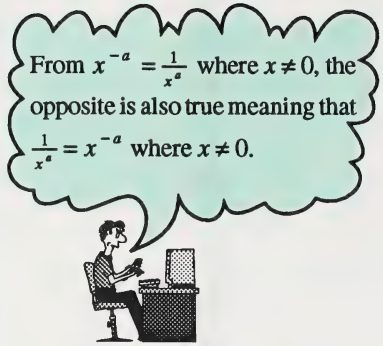
2^{-3}

Enter	Display
2	2
x^y or y^x	2
3	3
+/-	−3
=	0.125

$\frac{1}{2^3}$

Enter	Display
2	2
x^y or y^x	2
3	3
=	8
Inv or 2nd F	8
1/x	0.125

Since $2^{-3} = 0.125$ and $\frac{1}{2^3} = 0.125$, then $2^{-3} = \frac{1}{2^3}$.



The reciprocal of 6 is $\frac{1}{6}$ and the reciprocal of x is $\frac{1}{x}$. Likewise, the reciprocal of $-\frac{1}{8} = -8$ and of $-\frac{1}{y} = -y$.

Example 16

Use your calculator to show that $5^{-2} = \frac{1}{5^2}$.

Solution:

5^{-2}

Enter	Display
5	5
x^y or y^x	5
2	2
+/-	−2
=	0.04

$\frac{1}{5^2}$

Enter	Display
5	5
x^y or y^x	5
2	2
=	25
Inv or 2nd F	25
1/x	0.04

Since $5^{-2} = 0.04$ and $\frac{1}{5^2} = 0.04$, then $5^{-2} = \frac{1}{5^2}$.

The examples support the rule about powers with negative exponents which states that

$x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^a} = x^{-a}$.

Example 17

Convert the following to expressions having powers with positive exponents.

- 2^{-15}

Solution:

$$2^{-15} = \frac{1}{2^{15}}$$

- $(-50)^{-12}$

  Solution:

$$(-50)^{-12} = \frac{1}{(-50)^{12}}$$

- 2^{-1}

  Solution:

$$2^{-1} = \frac{1}{2^1}$$

The rules of order of operations must be followed when several operations are involved. Use BEDMAS when evaluating expressions such as those in Example 18.

1. Brackets
2. Exponents
3. Division
4. Multiplication
5. Addition
6. Subtraction

Look at the following examples.

Example 18

Evaluate the following.

- $\dfrac{2^{-1}}{\left(2^{-2}+3^{-1}\right)}$

  Solution:

$$\frac{2^{-1}}{\left(2^{-2}+3^{-1}\right)} = \frac{\frac{1}{2}}{\left(\frac{1}{2^2}+\frac{1}{3}\right)}$$

$$= \frac{\frac{1}{2}}{\left(\frac{1}{4}+\frac{1}{3}\right)}$$

$$= \frac{\frac{1}{2}}{\left(\frac{3+4}{12}\right)}$$

$$= \frac{\frac{1}{2}}{\frac{7}{12}}$$

$$= \frac{1}{2} + \frac{7}{12}$$

$$= \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{12}^{6}}{7}$$

$$= \frac{6}{7}$$

- $\frac{10^{-1}}{\left(10^{-2}+1\right)}$

Solution:

$$\frac{10^{-1}}{\left(10^{-2}+1\right)} = \frac{\frac{1}{10}}{\left(\frac{1}{10^2}+1\right)}$$

$$= \frac{\frac{1}{10}}{\left(\frac{1}{100}+1\right)}$$

$$= \frac{\frac{1}{10}}{\frac{101}{100}}$$

$$= \frac{1}{10} \div \frac{101}{100}$$

$$= \frac{1}{\cancel{10}_{1}} \times \frac{\cancel{100}^{10}}{101}$$

$$= \frac{10}{101}$$

- $10^0 + 5^0 + 2^{-1}$

Solution:

$$10^0 + 5^0 + 2^{-1} = 1 + 1 + \frac{1}{2}$$

$$= 2\frac{1}{2}$$

You now understand powers with negative exponents. So far you have used the Product Law and Quotient Law with only positive exponents. They also apply when you work with negative and zero exponents.

Example 19

Evaluate the following.

- $5^4 \times 5^{-3}$

Solution:

$$5^4 \times 5^{-3} = 5^{4+(-3)}$$

$$= 5^{4-3}$$

$$= 5^1 \text{ or } 5$$

- $3^{-10} \times 3^8$

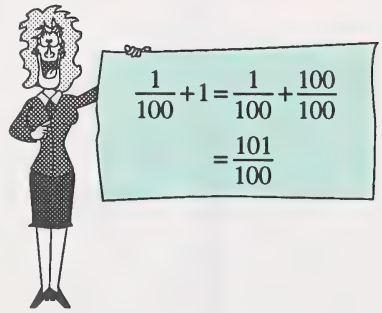
Solution:

$$3^{-10} \times 3^8 = 3^{-10+8}$$

$$= 3^{-2}$$

$$= \frac{1}{3^2}$$

$$= \frac{1}{9}$$



- $2^5 + 2^6$

  Solution:

$$\begin{aligned} 2^5 + 2^6 &= 2^{5-6} \\ &= 2^{-1} \\ &= \frac{1}{2} \end{aligned}$$

- $50^3 + 50^0$

  Solution:

$$\begin{aligned} 50^3 + 50^0 &= 50^{3-0} \\ &= 50^3 \\ &= 125\ 000 \end{aligned}$$

Try some questions to see if you understand.

1. Evaluate the following. For d leave the answer as a fraction or round the answer to three decimal places. For e and f give the answer as a fraction and as a decimal number.

   a. $(-5)^0$ b. 7^0

   c. 125^0 d. 3^{-3}

   e. 5^{-2} f. 10^{-5}

2. Use powers with positive exponents to express the following.

   a. 9^{-12} b. 54^{-3}

3. Evaluate the following.

   a. $2^{-1} + 3^{-1}$ b. $5^0 + 2^{-2} + 5^{-1}$

   c. $\dfrac{1}{\left(2^{-1} + 4^{-1}\right)}$ d. $\dfrac{2^2 \times 3^2}{3^3}$

   e. $\left(5^3 + 5^5\right) \times 2^2$ f. $25^{15} \times 25^{-15}$

For solutions to **Activity 4**, turn to the **Appendix**, **Topic 4**.

If you had difficulty with this topic, go to the **Extra Help** section at the end of this topic. If you want more challenges, go to the **Extensions** section.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

Powers with negative and zero exponents may still seem a little confusing. You saw how the quotient law gave a way of interpreting zero and negative exponents, but you may want more evidence to show their meaning.

Look at the following pattern.

$2^5 = 32$

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = ?$

Can you name the next entry? You can see that the numbers diminish by a factor of $\frac{1}{2}$ as you move down step by step. For example $32 \times \frac{1}{2} = 16$, $16 \times \frac{1}{2} = 8$, and so on. To get the missing entry, you would take $2 \times \frac{1}{2}$ which is 1. This means that $2^0 = 1$.

What happens to the values as you continue to diminish the exponents?

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = ?$

If you follow the same pattern, the next entry will be $1 \times \frac{1}{2}$ which is $\frac{1}{2}$.

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = ?$

Continuing in this way, the next entry will be $\frac{1}{2}\times\frac{1}{2}$ which is $\frac{1}{4}$ or $\frac{1}{2^2}$.

.

.

.

$2^{-1}=\frac{1}{2}$

$2^{-2}=\frac{1}{4}=\frac{1}{2^2}$

$2^{-3}=?$

The next entry will be $\frac{1}{4}\times\frac{1}{2}$ which is $\frac{1}{8}$ or $\frac{1}{2^3}$.

.

.

.

$2^{-2}=\frac{1}{4}=\frac{1}{2^2}$

$2^{-3}=\frac{1}{8}=\frac{1}{2^3}$

This pattern suggests that $2^0=1$ and $2^{-a}=\frac{1}{2^a}$.

Will you get similar results when another base is used? Look at what happens in this case.

$3^4=81$

$3^3=27$

$3^2=9$

$3^1=3$

Here the values diminish by a factor of $\frac{1}{3}$ each time you move down one step. The next entry would be $3\times\frac{1}{3}=1$.

$3^4=81$

$3^3=27$

$3^2=9$

$3^1=3$

$3^0=1$

$3^{-1}=?$

The next entries would be $1\times\frac{1}{3}=\frac{1}{3}$, $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$, $\frac{1}{9}\times\frac{1}{3}=\frac{1}{27}$, $\frac{1}{27}\times\frac{1}{3}=\frac{1}{81}$, and so on.

$3^4 = 81$
$3^3 = 27$
$3^2 = 9$
$3^1 = 3$
$3^0 = 1$
$3^{-1} = \frac{1}{3}$
$3^{-2} = \frac{1}{9} = \frac{1}{3^2}$
$3^{-3} = \frac{1}{27} = \frac{1}{3^3}$
$3^{-4} = \frac{1}{81} = \frac{1}{3^4}$

This pattern suggests that $3^0 = 1$ and $3^{-a} = \frac{1}{3^a}$.
Consider the pattern when the base is 5.

$5^4 = 625$
$5^3 = 125$
$5^2 = 25$
$5^1 = 5$
$5^0 = 1$
$5^{-1} = \frac{1}{5}$
$5^{-2} = \frac{1}{25}$
$5^{-3} = \frac{1}{125}$
$5^{-4} = \frac{1}{625}$

Here again the power with the zero exponent equals 1 and the power with the negative exponent is equal to the reciprocal of the same power with the exponent made positive.

The results, $x^0 = 1$ and $x^{-a} = \frac{1}{x^a}$ for $x \neq 0$ should now be clearer to you.

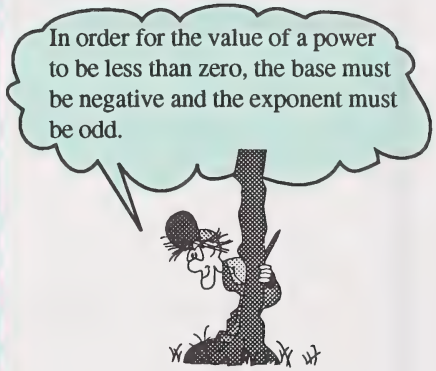
Notice that a negative exponent does not necessarily mean the power takes on a negative value. For example,

$5^{-3} = \frac{1}{15^3}$ but $(-5)^3 = (-5)\times(-5)\times(-5)$
$= \frac{1}{125}$ $= -125$

Try the following questions. Evaluate the following.

1. 2^{-2}
2. 3^0
3. 15^0
4. 1^0
5. 3^{-1}
6. 5^{-1}
7. 2^{-3}
8. 10^{-4}
9. 1000^0

For solutions to **Extra Help**, turn to the **Appendix, Topic 4**.



Extensions

You have seen how you can use short cuts by applying the Quotient Law and the Product Law. Are there other shortcuts?

Look at $(2^3)^2$. You could say that this is a power of a power.

$$(2^3)^2 = 8^2$$
$$= 64$$

Do you recognize that $2^6 = 64$? It is! You can conclude that $(2^3)^2 = 2^6$.

It seems that the exponents are multiplied to get the new exponent.

Check another case with the calculator. See if $(3^5)^3$ is equal to 3^{15}.

Evaluate 3^5.

Enter	Display
3	3
x^y or y^x	3
5	5
=	243

Therefore, $3^5 = 243$

Evaluate 243^3.

Enter	Display
243	243
x^y or y^x	243
3	3
=	14348907

This tells you that $(3^5)^3 = (243)^3 = 14\,348\,907$.

Evaluate 3^{15}.

Enter	Display
3	3
x^y or y^x	3
15	15
=	14348907

Thus, $3^{15} = 14\,348\,907$.

You can see that 3^{15} and $(3^5)^3$ have the same value. This means that $(3^5)^3 = 3^{15}$.

So far you have seen that exponents multiply for powers of powers. Will this be demonstrated by putting the powers in expanded form?

$$\begin{aligned}\left(2^4\right)^3 &= (2\times2\times2\times2)^3\\ &= (2\times2\times2\times2)\times(2\times2\times2\times2)\times(2\times2\times2\times2)\\ &= 2^{12}\end{aligned}$$

In the final expanded form, there are three groups of four factors. In total there are $3\times4=12$ factors.

$$(2\times2\times2\times2)\times(2\times2\times2\times2)\times(2\times2\times2\times2)=2^{12}$$

This means that $\left(2^4\right)^3 = 2^{12}$. Again exponents multiply in this case.

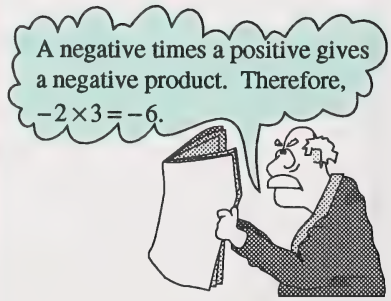
Consider the evaluation of another power of a power by putting it into expanded form.

$$\begin{aligned}\left(7^3\right)^5 &= (7\times7\times7)^5\\ &= (7\times7\times7)\times(7\times7\times7)\times(7\times7\times7)\times(7\times7\times7)\times(7\times7\times7)\\ &= 7^{15}\end{aligned}$$

What if negative exponents are involved, as in $\left(2^{-2}\right)^3$?

$$\begin{aligned}\left(2^{-2}\right)^3 &= \left(\frac{1}{2\times2}\right)^3\\ &= \frac{1}{2\times2}\times\frac{1}{2\times2}\times\frac{1}{2\times2}\\ &= \frac{1}{2\times2\times2\times2\times2\times2}\\ &= \frac{1}{2^6}\\ &= 2^{-6}\end{aligned}$$

Here again the exponents multiply.



It turns out that to evaluate a **power of a power**, the exponents are **multiplied** to give the exponent in the answer. This holds even when negative exponents are involved. However, when the exponenet is negative, the base cannot be zero.

You can express these results better as follows.

Power of a Power Law

$$\left(x^{a}\right)^{b} = x^{a \times b}$$
$$= x^{ab}$$

Example 20

Express each of the following as a power with a single exponent.

- $\left(5^{4}\right)^{3}$

  Solution:

  $$\left(5^{4}\right)^{3} = 5^{12}$$

- $\left(6^{8}\right)^{2}$

  Solution:

  $$\left(6^{8}\right)^{2} = 6^{16}$$

- $\left(2^{-4}\right)^{2}$

  Solution:

  $$\left(2^{-4}\right)^{2} = 2^{-8}$$

- $\left(4^{2}\right)^{-1} = 4^{-2}$

  Solution:

  $$\left(4^{2}\right)^{-1} = 4^{-2}$$

- $\left(10^{10}\right)^{0}$

  Solution:

  $$\left(10^{10}\right)^{0} = 10^{0}$$

Apply the Power of a Power Law to the following questions.

1. Express each of the following as a power with a single exponent.

   a. $\left(10^3\right)^5$ b. $\left(2^9\right)^2$

   c. $\left(11^5\right)^2$ d. $\left(5^2\right)^{-3}$

2. Evaluate the following.

   a. $\left(5^3\right)^2$ b. $\left(10^{-1}\right)^2$

   c. $\left[(-2)^2\right]^3$ d. $\left(3^{-2}\right)^{-1}$

For solutions to **Extensions**, turn to the **Appendix, Topic 4.**

So far you have dealt with powers having integers as bases. However, powers with any rational bases can be evaluated by applying the laws of exponents as well.

In a power, the exponent tells how many times the base is a factor in expanded multiplication form.

Example 21

Express the following in expanded multiplication form.

- $\left(\frac{3}{4}\right)^5$

Solution:

$$\left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)$$

- $(1.5)^6$

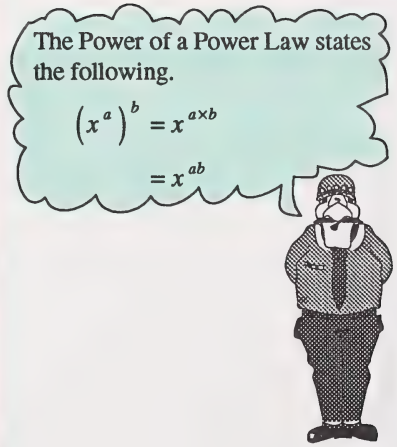
Solution:

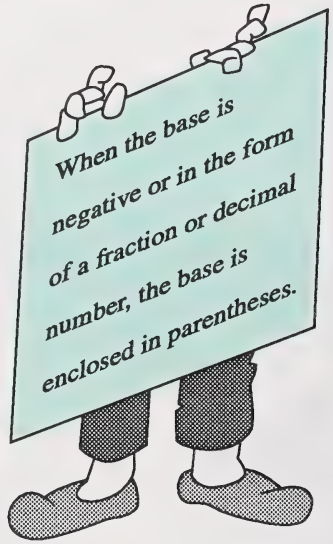
$$(1.5)^6 = (1.5)\times(1.5)\times(1.5)\times(1.5)\times(1.5)\times(1.5)$$

- $\left(-\frac{1}{2}\right)^4$

Solution:

$$\left(-\frac{1}{2}\right)^4 = \left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)$$





A power can be evaluated by simplifying the expanded form.

Example 22

Evaluate the following.

- $\left(\frac{1}{2}\right)^3$

Solution:

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$$
$$= \frac{1}{8}$$

- $\left(-\frac{3}{4}\right)^3$

Solution:

$$\left(-\frac{3}{4}\right)^3 = \left(-\frac{3}{4}\right)\times\left(-\frac{3}{4}\right)\times\left(-\frac{3}{4}\right)$$
$$= -\frac{3\times3\times3}{4\times4\times4}$$
$$= -\frac{27}{64}$$

- $(0.2)^4$

Solution:

$$(0.2)^4 = (0.2)\times(0.2)\times(0.2)\times(0.2)$$
$$= 0.0016$$

The x^y or y^x button of a calculator can be used to evaluate powers.

Example 23

Evaluate the following using your calculator. Round to six decimal places.

- $(1.000\,001)^{12}$

Solution:

Enter	Display
1.000001	1.000001
x^y or y^x	1.000001
12	12
=	1.000012

$\therefore (1.000\,001)^{12} = 1.000\,012$

- $\left(\frac{3}{4}\right)^{16}$

Solution:

Enter	Display
3	3
+	3
4	4
=	0.75
x^y or y^x	0.75
16	16
=	0.010022595

$\therefore \left(\frac{3}{4}\right)^{16} \doteq 0.010\,023$

The Product and Quotient Laws also apply to rationals.

Example 24

Express as a single power.

- $\left(\frac{3}{4}\right)^4 \times \left(\frac{3}{4}\right)^{14}$

Solution:

$$\left(\frac{3}{4}\right)^4 \times \left(\frac{3}{4}\right)^{14} = \left(\frac{3}{4}\right)^{4+14}$$
$$= \left(\frac{3}{4}\right)^{18}$$

- $(0.9)^{12} + (0.9)^{10}$

Solution:

$$(0.9)^{12} + (0.9)^{10} = (0.9)^{12-10}$$
$$= (0.9)^2$$

Remember and use the power laws to make your work easier.

A power with a zero exponent is equal to one. This holds for all rational bases except when the base is zero.

Example 25

Evaluate the following.

- $\left(\frac{13}{17}\right)^0$

  Solution:

  $$\left(\frac{13}{17}\right)^0 = 1$$

- $(12.825)^0 + (0.175)^0$

  Solution:

  $$(12.825)^0 + (0.175)^0 = 1 + 1$$
  $$= 2$$

A power with a negative exponent is equal to the reciprocal of the corresponding power with the exponent made positive. This rule is the same as earlier, but it is not restricted to integer bases only.

Example 26

Convert the following to expressions involving powers with positive exponents.

- $(0.125)^{-3}$

  Solution:

  $$(0.125)^{-3} = \frac{1}{(0.125)^3}$$

- $(1.01)^{-10}$

  Solution:

  $$(1.01)^{-10} = \frac{1}{(1.01)^{10}}$$

It is important to memorize the following number facts.

- $x^0 = 1$
- $0^0 =$ undefined
- $x^{-a} = \frac{1}{x^a}$

Now try the following questions.

3. Evaluate the following. Round to three decimal places where required. Use a calculator.

   a. $(1.05)^5$

   b. $\left(\frac{2}{5}\right)^3$

   c. $(3.14)^{21} \div (3.14)^{20}$

   d. $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2$

   e. $(0.005)^{-2}$

   f. $(0.125)^{-5}$

   g. $\left(\frac{2}{3}\right)^0$

   h. $\left(\frac{9}{16}\right)^{-3}$

4. Express the following as a single power.

   a. $\left(-\frac{2}{3}\right)^9 \times \left(-\frac{2}{3}\right)^9$

   b. $\left(\frac{1}{2}\right)^{14} \times \left(\frac{1}{2}\right)^3$

   c. $\left(\frac{1}{2}\right)^{14} \div \frac{1}{2}$

   d. $(1.01)^{15} \div (1.01)^5$

   e. $\left(\frac{3}{11}\right)^4 \div \left(\frac{3}{11}\right)^6$

5. Use powers with positive exponents to express the following.

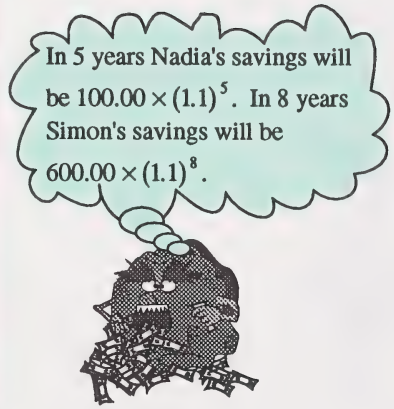
   a. $\left(\frac{7}{10}\right)^{-5}$

   b. $(12.75)^{-9}$

6. Nadia has money in a savings account. The interest rate is 10% annually. She keeps the interest in her account so every year her savings increase by a factor of 1.1. In five years, her account will grow by a factor of $(1.1)^5$. She initially deposited \$100.00 so at the end of five years she will have $\$100.00 \times (1.1)^5$. Calculate what her savings will be after five years and how much interest she will earn in that time. Give your answers to the nearest cent.

7. Simon has \$600.00 on deposit at 10% annually. He keeps the interest in a savings account. This way his savings increase by a factor of 1.1 each year. What will his savings in this account be after a period of eight years and how much interest will he earn in that time? Round your answers to the nearest cent.

For solutions to **Extensions**, turn to the **Appendix, Topic 4**.



Unit Summary

What You Have Learned

In this unit you learned about rational numbers. You found that they could be expressed using decimals and fractions. You learned how they could be added, subtracted, multiplied, and divided.

You learned about a special form for expressing portions of a whole by using percent notation. From there, you continued with squares and square roots.

You discovered how multiplication and division of powers led to addition and subtraction of exponents. You found out what happens when you have a power with a zero exponent, an exponent of one, and negative exponents.

After this unit, you are in a better position to understand expressions such as $\frac{2}{3}$ of a cup, 0.53 m, an interest rate of 10%, and $\sqrt{100}$. You can evaluate 2^5, 3^4, and similar expressions.

You are now ready to

complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1 Rational Numbers

Topic 2 Percent

Topic 3 Squares and Square Roots

Topic 4 Laws of Exponents

Review

1. a. -502 *N* *W* (*I*)

   b. 37 (*N*) (*W*) (*I*)

   c. 1 (*N*) (*W*) (*I*)

   d. 0 *N* (*W*) (*I*)

2. a.

Enter	Display
139	139
×	139
520	520
=	72280

   b.

Enter	Display
26124	26124
−	26124
11235	11235
=	14889

   c.

Enter	Display
5280	5280
+	5280
176	176
=	30

   d.

Enter	Display
340	340
+	340
1129	1129
+	1469
2670	2670
=	4139

3. a.
$$\begin{array}{r} {}^{1}87 \\ {}^{2}163 \\ {}^{1}1314 \\ +\ 2991 \\ \hline 4555 \end{array}$$

   b.
$$\begin{array}{r} 97500 \\ -87502 \\ \hline 9998 \end{array}$$

   c.
$$\begin{array}{r} 126 \\ \times 48 \\ \hline 1008 \\ 504 \\ \hline 6048 \end{array}$$

   d.
$$\begin{array}{r} 43 \\ 12\overline{)516} \\ 48 \\ \hline 36 \\ 36 \\ \hline 0 \end{array}$$

4. a. $2 = (2)$

   $14 = 7 \times 2$

   $49 = (7 \times 7)$

   L.C.M. $= 2 \times 7 \times 7$

   $= 98$

   or

2	2	14	49
7	1	7	49
	1	1	7

   L.C.M. $= 2 \times 7 \times 1 \times 1 \times 7$

   $= 98$

b. $18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$
L.C.M. $= 2 \times 3 \times 3 \times 5$
$= 90$

or

2	18 30
3	9 15
	3 5

L.C.M. $= 2 \times 3 \times 3 \times 5$
$= 90$

5. a. $36 = 2 \times 2 \times 3 \times 3$
$60 = 2 \times 2 \times 3 \times 5$
G.C.F. $= 2 \times 2 \times 3$
$= 12$

or

6	36 60
2	6 10
1	3 5
	3 5

G.C.F. $= 6 \times 2 \times 1$
$= 12$

b. $14 = 2 \times 7$
$49 = 7 \times 7$
$98 = 2 \times 7 \times 7$
G.C.F. $= 7$

or

7	14 49 98
1	2 7 14
	2 7 14

G.C.F. $= 7 \times 1$
$= 7$

If you had trouble with the Review, go to **Mathematics 9, Unit 2.**

Exploring Topic 1

Activity 1

Understand the relationship between integers and rational numbers.

1. a. $\frac{27}{3} = 9$ (integer) b. $\frac{-31}{1} = -31$ (integer)

c. $\frac{36}{18} = 2$ (integer) d. $\frac{-42}{7} = -6$ (integer)

e. $\frac{-35}{7} = -5$ (integer) f. $\frac{-42}{14} = -3$ (integer)

g. $\frac{51}{17} = 3$ (integer) h. $\frac{-54}{3} = -18$ (integer)

2. a. $\frac{28}{7} = 4$ b. $\frac{26}{13} = 2$ c. $\frac{39}{3} = 13$

d. $\frac{52}{4} = 13$ e. $\frac{51}{17} = 3$ f. $\frac{57}{19} = 3$

g. $\frac{-60}{4} = -15$ h. $\frac{24}{-3} = -8$

3. All integers can be written over the number 1. This makes them all rational numbers. Rational numbers are formed by placing an integer over an integer. Rationals do not always produce integers. Examples of integers that are rationals are 2, −13, 27, which may be written as $\frac{2}{1}$, $\frac{-13}{1}$, $\frac{27}{1}$ respectively.

   Example of rationals that are not integers are: $\frac{22}{7}$, $\frac{2}{3}$, $-4\frac{1}{3}$, etc.

4. A number is a rational number if it can be put into the form of $\frac{a}{b}$ where a and b are integers, and b is not equal to zero.

   a. $25 = \frac{25}{1}$ b. $-1\frac{3}{4} = \frac{-7}{4}$ c. $7\frac{1}{3} = \frac{22}{3}$

   d. $-31 = \frac{-31}{1}$ e. $1\frac{9}{7} = \frac{16}{7}$ f. $34 = \frac{34}{1}$

   g. $3\frac{1}{7} = \frac{22}{7}$ h. $\frac{36}{48} = \frac{3}{4}$

5. No, this cannot be done. It is impossible to write even one integer that is not a rational number because all integers are rational numbers since they can be expressed as fractions using one as a denominator.

Activity 2

Perform addition and subtraction involving fractions.

1. a. $\frac{1}{2} - \frac{5}{12} = \frac{1\times6}{2\times6} - \frac{5}{12}$

$= \frac{6}{12} - \frac{5}{12}$

$= \frac{1}{12}$

b. $\frac{9}{16} + \frac{3}{4} = \frac{9}{16} + \frac{3\times4}{4\times4}$

$= \frac{9}{16} + \frac{12}{16}$

$= \frac{21}{16}$

$= 1\frac{5}{16}$

c. $\frac{1}{2} + \frac{7}{10} = \frac{1\times5}{2\times5} + \frac{7}{10}$

$= \frac{5}{10} + \frac{7}{10}$

$= \frac{12}{10}$

$= 1\frac{2}{10}$

$= 1\frac{1}{5}$

2. a. $\frac{2}{3}+\frac{3}{4}=\frac{2\times4}{3\times4}+\frac{3\times3}{4\times3}$

$=\frac{8}{12}+\frac{9}{12}$

$=\frac{17}{12}$

$=1\frac{5}{12}$

b. $\frac{2}{5}-\frac{2}{15}=\frac{2\times3}{5\times3}-\frac{2}{15}$

$=\frac{6}{15}-\frac{2}{15}$

$=\frac{4}{15}$

c. $\frac{3}{4}-\frac{5}{8}=\frac{3\times2}{4\times2}-\frac{5}{8}$

$=\frac{6}{8}-\frac{5}{8}$

$=\frac{1}{8}$

3. a. $3\frac{1}{2}+1\frac{1}{3}=\frac{7}{2}+\frac{4}{3}$

$=\frac{7\times3}{2\times3}+\frac{4\times2}{3\times2}$

$=\frac{21}{6}+\frac{8}{6}$

$=\frac{29}{6}$

$=4\frac{5}{6}$

or

$3\frac{1}{2}=3\frac{3}{6}$

$+1\frac{1}{3}=1\frac{2}{6}$

$4\frac{5}{6}$

b. $5\frac{1}{2}+\frac{7}{8}=\frac{11}{2}+\frac{7}{8}$

$=\frac{11\times4}{2\times4}+\frac{7}{8}$

$=\frac{44}{8}+\frac{7}{8}$

$=\frac{51}{8}$

$=6\frac{3}{8}$

or

$5\frac{1}{2}=5\frac{4}{8}$

$+\frac{7}{8}=\frac{7}{8}$

$5\frac{11}{8}=5+\frac{8}{8}+\frac{3}{8}$

$=5+1+\frac{3}{8}$

$=6\frac{3}{8}$

c. $16+8\frac{1}{2}=\frac{16}{1}+\frac{17}{2}$

$=\frac{16\times2}{1\times2}+\frac{17}{2}$

$=\frac{32}{2}+\frac{17}{2}$

$=\frac{49}{2}$

$=24\frac{1}{2}$

or

16

$+8\frac{1}{2}$

$24\frac{1}{2}$

4. a. $3\frac{1}{2}-2=\frac{7}{2}-\frac{2}{1}$ or $3\frac{1}{2}$

$=\frac{7}{2}-\frac{2\times2}{1\times2}$ $\quad -2$

$=\frac{7}{2}-\frac{4}{2}$ $\quad 1\frac{1}{2}$

$=\frac{3}{2}$

$=1\frac{1}{2}$

b. $3\frac{2}{3}-2\frac{5}{6}=\frac{11}{3}-\frac{17}{6}$ or $3\frac{2}{3}=3\frac{4}{6}=2\frac{10}{6}$

$=\frac{11\times2}{3\times2}-\frac{17}{6}$ $\quad -2\frac{5}{6}=2\frac{5}{6}=2\frac{5}{6}$

$=\frac{22}{6}-\frac{17}{6}$ $\quad \frac{5}{6}$

$=\frac{5}{6}$

c. $6-3\frac{7}{8}=\frac{6\times8}{1\times8}-\frac{31}{8}$ or $6=5\frac{8}{8}$

$=\frac{48}{8}-\frac{31}{8}$ $\quad -3\frac{7}{8}=3\frac{7}{8}$

$=\frac{17}{8}$ $\quad 2\frac{1}{8}$

$=2\frac{1}{8}$

5. a. $2\,|\,2\ 4\ 8$

$2\,|\,1\ 2\ 4$

$1\ 1\ 2$

L.C.D. $=2\times2\times1\times1\times2$

$=8$

The L.C.D. for $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ is 8.

b. $2\,|\,2\ 3\ 4$

$1\ 3\ 2$

L.C.D. $=2\times1\times3\times2$

$=12$

The L.C.D. for $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ is 12.

c. $1\,|\,13\ 11$

$13\ 11$

L.C.D. $=1\times13\times11$

$=143$

The L.C.D. for $\frac{8}{13}$ and $\frac{9}{11}$ is 143.

d. $2\,|\,5\ 6\ 8$

$5\ 3\ 4$

L.C.D. $=2\times5\times3\times4$

$=120$

The L.C.D. for $\frac{1}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ is 120.

Activity 3

Perform multiplication and division involving fractions.

1. a. $\frac{2}{5}\times\frac{3}{4}=\frac{2\times3}{5\times4}$

$=\frac{6}{20}$

$=\frac{3}{10}$

b. $\frac{3}{7}\times\frac{3}{10}=\frac{3\times3}{7\times10}$

$=\frac{9}{70}$

c. $1\frac{5}{9}\times2\frac{1}{3}=\frac{14}{9}\times\frac{7}{3}$

$=\frac{14\times7}{9\times3}$

$=\frac{98}{27}$

$=3\frac{17}{27}$

2. a. $2\frac{7}{10}\times6\frac{2}{3}=\frac{27}{10}\times\frac{20}{3}$

$=\frac{27\times20}{10\times3}$

$=\frac{540}{30}$

$=18$

b. $\frac{6}{5}\times3\frac{1}{2}=\frac{6}{5}\times\frac{7}{2}$

$=\frac{6\times7}{5\times2}$

$=\frac{42}{10}$

$=4\frac{2}{10}$

$=4\frac{1}{5}$

c. $7\div2\frac{1}{3}=\frac{7}{1}\div\frac{7}{3}$

$=\frac{7}{1}\times\frac{3}{7}$

$=\frac{7\times3}{1\times7}$

$=\frac{21}{7}$

$=3$

3. a. $5\div1\frac{1}{2}=\frac{5}{1}\div\frac{3}{2}$

$=\frac{5}{1}\times\frac{2}{3}$

$=\frac{10}{3}$

$=3\frac{1}{3}$

b. $6\frac{1}{2}\div3=\frac{13}{2}\div\frac{3}{1}$

$=\frac{13}{2}\times\frac{1}{3}$

$=\frac{13}{6}$

$=2\frac{1}{6}$

c. $18\div27=\frac{18}{1}\div\frac{27}{1}$

$=\frac{18}{1}\times\frac{1}{27}$

$=\frac{18}{27}$

$=\frac{2}{3}$

4. Height of books = 45 cm.
Thickness of each book = $2\frac{1}{2}$ cm.

$$45 \div 2\frac{1}{2} = \frac{45}{1} \div \frac{5}{2}$$
$$= \frac{45}{1} \times \frac{2}{5}$$
$$= \frac{90}{5}$$
$$= 18$$

There are 18 books in the pile.

5. Capacity of tank = 72 litres.
When finding a fraction of a number, muliply the fraction and the number.

$$\frac{3}{4} \text{ of } 72 = \frac{3}{4} \times 72$$
$$= \frac{216}{4}$$
$$= 54$$

A tank that holds 72 litres when full will hold 54 litres when $\frac{3}{4}$ full.

Activity 4

Perform addition and subtraction involving decimals.

1. a. 23.014 + 8.390 + 1.400 = 32.804
b. 0.530 + 0.900 + 0.008 = 1.438
c. 164.4300 + 0.6970 + 8.5654 = 173.6924

2. \$3.85 + \$5.05 + \$4.95 + \$3.50 + \$5.95 = \$23.30

The workers would earn a total of \$23.30 per hour.

3. a. 3.157 − 2.500 = 0.657
b. 1.0250 − 0.9800 = 0.0450
c. 165.000 − 95.625 = 69.375

4. a. 62.455 − 29.090 = 33.365
b. 1.900 − 0.002 = 1.898
c. 170.050 − 78.651 = 91.399

5. The perimeter of the farmer's field is
21.9 m + 25.05 m + 101.5 m + 9.25 m + 65 m.

$$\begin{array}{r} 21.90\text{ m} \\ 25.05\text{ m} \\ 101.50\text{ m} \\ 9.25\text{ m} \\ +\ 65.00\text{ m} \\ \hline 222.70\text{ m} \end{array}$$

The perimeter of the field is 222.70 m.

6. a. $\begin{array}{r} 18.006 \\ -11.900 \\ \hline 6.106 \end{array}$ b. $\begin{array}{r} 0.9034 \\ -0.0129 \\ \hline 0.8905 \end{array}$ c. $\begin{array}{r} 343.080 \\ -245.208 \\ \hline 97.872 \end{array}$

Activity 5

Perform multiplication and division of decimals.

1. a. $0.025 \times 0.135 = 0.003\,375$ b. $2.75 \times 3.2 = 8.800$ or 8.8

c. $5.7 \times 0.63 = 3.591$ d. $3.3 \times 4.5 = 14.85$

e. $1.25 \times 33.6 = 42.000$ or 42 f. $0.413 \times 17.5 = 7.2275$

2. a. $\begin{array}{r} 1.35 \\ \times\ 6.23 \\ \hline 8.4105 \end{array}$ b. $\begin{array}{r} 5.25 \\ \times\ 0.75 \\ \hline 3.9375 \end{array}$ c. $\begin{array}{r} 0.25 \\ \times\ 0.125 \\ \hline 0.03125 \end{array}$

d. $\begin{array}{r} 28.7 \\ \times\ 16.15 \\ \hline 463.505 \end{array}$ e. $\begin{array}{r} 276.4 \\ \times\ 0.825 \\ \hline 228.0300 \text{ or } 228.03 \end{array}$

3. a. $1.65 \div 12$

$$\begin{array}{r} 0.1375 \\ 12\overline{)1.6500} \\ 12 \\ \hline 45 \\ 36 \\ \hline 90 \\ 84 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

$\therefore 1.65 \div 12 = 0.1375$

$\doteq 0.138$ (to three decimal places)

b. $0.087 \div 2.8 = \frac{0.87}{28}$

$$
\begin{array}{r}
0.03107 \\
28\overline{)0.87000} \\
84 \\
\hline
30 \\
28 \\
\hline
20 \\
0 \\
\hline
200 \\
196 \\
\hline
4
\end{array}
$$

$\therefore 0.087 \div 2.8 = 0.03107\ldots$

$\doteq 0.031$ (to three decimal places)

c. $52.13 \div 2.08 = \frac{5213}{208}$

$$
\begin{array}{r}
25.0625 \\
208\overline{)5213.0000} \\
4160 \\
\hline
1053 \\
1040 \\
\hline
1300 \\
1248 \\
\hline
520 \\
416 \\
\hline
1040 \\
1040 \\
\hline
0
\end{array}
$$

$\therefore 52.13 \div 2.08 = 25.0625$

$\doteq 25.063$ (to three decimal places)

4. a. $735.45 \div 25.7 = \dfrac{7354.5}{257}$

$$
\begin{array}{r}
28.61673 \\
257\overline{)7354.50000} \\
514 \\
\hline
2214 \\
2056 \\
\hline
1585 \\
1542 \\
\hline
430 \\
257 \\
\hline
1730 \\
1542 \\
\hline
1880 \\
1799 \\
\hline
810 \\
771 \\
\hline
39
\end{array}
$$

$\therefore 735.45 \div 25.7 \doteq 28.6167$

b. $49.0 \div 2.401 = \dfrac{49\,000}{2401}$

$$
\begin{array}{r}
20.40816 \\
2401\overline{)49000.00000} \\
4802 \\
\hline
980 \\
0 \\
\hline
9800 \\
9604 \\
\hline
1960 \\
0 \\
\hline
19600 \\
19208 \\
\hline
3920 \\
2401 \\
\hline
15190 \\
14406 \\
\hline
784
\end{array}
$$

$\therefore 49.0 \div 2.401 \doteq 20.4082$

c. $0.032 \div 10.24 = \frac{3.2}{1024}$

$$
\begin{array}{r}
0.00312 \\
1024\overline{)3.20000} \\
0 \\
\hline
320 \\
0 \\
\hline
3200 \\
3072 \\
\hline
1280 \\
1024 \\
\hline
2560 \\
2048 \\
\hline
572
\end{array}
$$

$\therefore 0.032 \div 10.24 \doteq 0.0031$

5. a. $14.44 \div 1.44 = \frac{1444}{144}$

$$
\begin{array}{r}
10.0277 \\
144\overline{)1444.0000} \\
144 \\
\hline
04 \\
0 \\
\hline
40 \\
0 \\
\hline
400 \\
288 \\
\hline
1120 \\
1008 \\
\hline
1120 \\
1008 \\
\hline
112
\end{array}
$$

$\therefore 14.44 \div 1.44 \doteq 10.028$

b.
$$\begin{array}{r} 5.6 \\ \times\ 17.1 \\ \hline 56 \\ 392 \\ 56 \\ \hline 95.76 \end{array}$$

$\therefore 5.6 \times 17.1 = 95.76$

c.
$$\begin{array}{r} 48.0 \\ \times\ 0.66 \\ \hline 2880 \\ 2880 \\ \hline 31.680 \end{array}$$

$\therefore 48.0 \times 0.66 = 31.68$

6. Side of chess board = 38.02 cm

$A = s^2$

$= 38.02 \text{ cm} \times 38.02 \text{ cm}$

$= 1445.5204 \text{ cm}^2$

The area of the chess board is 1445.5204 cm^2.

7. Area of rectangular field = 772.65 m^2
Known side of rectangular field = 25.5 m

$$\begin{array}{r} 30.3 \\ 255\overline{)7726.5} \\ 765 \\ \hline 765 \\ 765 \\ \hline 0 \end{array}$$

or

Other side of retangular field = $772.65 \text{ m}^2 + 25.5 \text{ m}$

$$= \frac{772.65 \text{ m}^2}{25.5 \text{ m}}$$

$$= \frac{7726.5 \text{ m}^2}{255 \text{ m}}$$

$= 30.3 \text{ m}$

The other side is 30.3 m in length.

Activity 6

Round and estimate with decimals.

1. When rounding to the nearest thousand, consider the fourth digit to the left of the decimal point. For example, 9443.6071 becomes 9000. When rounding to the nearest thousandth, consider the third digit after the decimal. For example, 9443.6071 becomes 9443.607.

2. Rounding to the nearest

	tenth	one
a. 491.99	492.0	492
b. 222.95	223.0	223
c. 95.59	96.6	96
d. 45.08	45.1	45
e. 19.473	19.5	19
f. 9.849	9.8	10

Notice how the tenths all have one decimal place.

3. The nearerst

	dollar	cent
a. $219.09	$219.00	$219.09
b. $0.956	$1.00	$0.96
c. $250.49	$250.00	$250.49
d. $111.974	$112.00	$112.97
e. $343.14	$343.00	$343.14

4. a. $301\times497=300\times500$
$=150\,000$

b. $949\times101=950\times100$
$=950\,000$

c. $0.91\times0.502=0.9\times0.5$
$=0.45$

d. $298.4\times209.87=300\times210$
$=63\,000$

e. $3.05\times15.004=3\times15$
$=45$

Note how each is rounded and then multiplied.

5. Round \$9.58 is to \$10.00. Estimate $9\times10\times5=\$450.00$. Therefore, \$450.00 is the closest estimate.

6. Round 430 pages to 400 pages, 21 pages to 20 pages, and 49 minutes to 50 minutes. Estimate $\frac{400}{20}\times50=1000$. Therefore, 1000 minutes is the closest estimate.

7. a. $\dfrac{9.8\times19.7\times51.01}{2.1\times4.9}=\dfrac{10\times20\times50}{2\times5}$
$=\dfrac{10\,000}{10}$
$=1000$

b. $\frac{49\times501}{2.5}=\frac{50\times500}{2.5}$

$=\frac{500\times500}{25}$

$=\frac{250\ 000}{25}$

$=10\ 000$

c. $39\frac{30}{31}\times30\frac{1}{29}=40\times30$

$=1200$

Activity 7

Change fractions to decimals.

1. a. $\frac{1}{2}=\frac{1\times5}{2\times5}$ or $2\overline{)1.0}$ 0.5

$=\frac{5}{10}$ 10

$=0.5$ 0

b. $\frac{1}{4}=\frac{1\times25}{4\times25}$ or $4\overline{)1.00}$ 0.25

$=\frac{25}{100}$ 8

$=0.25$ 20

0

c. $\frac{3}{5}=\frac{3\times2}{5\times2}$ or $5\overline{)3.0}$ 0.6

$=\frac{6}{10}$ 30

$=0.6$ 0

d. $\frac{3}{4}=\frac{3\times25}{4\times25}$ or $4\overline{)3.00}$ 0.75

$=\frac{75}{100}$ 28

$=0.75$ 20

20

0

e. $\frac{4}{5}=\frac{4\times20}{5\times20}$ or $5\overline{)4.0}$ 0.8

$=\frac{80}{100}$ 40

$=0.8$ 0

2. a. $\frac{1}{8} = 1 \div 8$

$$
\begin{array}{r}
0.125 \\
8\overline{)1.000} \\
\underline{8} \\
20 \\
\underline{16} \\
40 \\
\underline{40} \\
0
\end{array}
$$

$\therefore \frac{1}{8} = 0.125$

b. $2\frac{1}{2} = 2.5$ since $2\frac{1}{2} = \frac{5}{2}$

$= 2.5$

$$
\begin{array}{r}
2.5 \\
2\overline{)5.0} \\
\underline{4} \\
10 \\
\underline{10} \\
0
\end{array}
$$

c. $7\frac{1}{4} = \frac{29}{4}$

$= \frac{29 \times 25}{4 \times 25}$

$= \frac{725}{100}$

$= 7.25$

d. $1\frac{7}{8} = \frac{15}{8}$

$= \frac{15 \times 125}{8 \times 125}$

$= \frac{1875}{1000}$

$= 1.875$

e. $2\frac{2}{5} = \frac{12}{5}$

$= \frac{12 \times 2}{5 \times 2}$

$= \frac{24}{10}$

$= 2.4$

3. a. $4\frac{1}{3} = \frac{13}{3}$

$$
\begin{array}{r}
4.333 \\
3\overline{)13.000} \\
\underline{12} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
1
\end{array}
$$

Therefore, $4\frac{1}{3} \doteq 4.333$ rounded to three decimal places.

b. $3\frac{1}{7} \doteq 3.143$ rounded to three decimal places.

c. $1\frac{5}{11} \doteq 1.455$ rounded to three decimal places.

d. $\frac{8}{9} \doteq 0.889$ rounded to three decimal places.

e. $2\frac{5}{6} \doteq 2.833$ rounded to three decimal places.

4. a. $\frac{0}{5} = 0$

b. $\frac{83}{1000} = 0.083$

c. $1\frac{7}{20} = \frac{27}{20}$
$= 1.35$

d. $\frac{23}{40} = 0.575$

e. $\frac{273}{100} = 2.73$

5. a. $17 = 17.0$

b. $\frac{5}{0}$ cannot be expressed as a decimal because division by zero is not possible or is undefined.

c. $2^3 = 2 \times 2 \times 2$
$= 8$
$= 8.0$

d. $\frac{2.5}{4} = \frac{25}{40}$

$= \frac{625}{1000}$

$= 0.625$

This can also be done by long division.

e. $\frac{0}{0}$ cannot be expressed as a decimal because division by zero is not possible or is undefined.

Activity 8

Change decimals to fractions.

1. a. $0.6 = \frac{6}{10}$

$= \frac{3}{5}$

b. $0.25 = \frac{25}{100}$

$= \frac{1}{4}$

c. $0.75 = \frac{75}{100}$

$= \frac{3}{4}$

d. $0.1 = \frac{1}{10}$

e. $1.5 = 1\frac{5}{10}$

$= 1\frac{1}{2}$

f. $3.2 = 3\frac{2}{10}$

$= 3\frac{1}{5}$

g. $2.25 = 2\frac{25}{100}$

$= 2\frac{1}{4}$

h. $7.3 = 7\frac{3}{10}$

i. $0.125 = \frac{125}{1000}$

$= \frac{1}{8}$

j. $1.375 = 1\frac{375}{1000}$

$= 1\frac{3}{8}$

k. $2.45 = 2\frac{45}{100}$

$= 2\frac{9}{20}$

l. $3.005 = 3\frac{5}{1000}$

$= 3\frac{1}{200}$

2. a. $3.125 = 3 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$

b. $0.3205 = 0 + \frac{3}{10} + \frac{2}{100} + \frac{0}{1000} + \frac{5}{10\,000}$

c. $434.525 = 400 + 30 + 4 + \frac{5}{10} + \frac{2}{100} + \frac{5}{1000}$

d. $47.0069 = 40 + 7 + \frac{0}{10} + \frac{0}{100} + \frac{6}{1000} + \frac{9}{10\,000}$

3. a. $0.02 = \frac{2}{100}$

$= \frac{1}{50}$

b. $2.035 = 2\frac{35}{1000}$

$= 2\frac{7}{200}$

c. $22.64 = 22\frac{64}{100}$

$= 2\frac{16}{25}$

d. $0.0005 = \frac{5}{10\,000}$

$= \frac{1}{2000}$

e. $1.1234 = 1\frac{1234}{10\,000}$

$= 1\frac{617}{5000}$

Activity 9

Perform calculations using the order of operations.

1. a. $(3^2 + 5) + 7 = (9 + 5) + 7$

$= 14 + 7$

$= 2$

b. $(5^2 - 3) \times 2 = (25 - 3) \times 2$

$= 22 \times 2$

$= 44$

c. $4^3 + 6^2 + 12 - 3^3 = 64 + 36 + 12 - 27$

$= 64 + 3 - 27$

$= 67 - 27$

$= 40$

d. $3(5 \times 3^2) - 35 = 3 \times 5 \times 3^2 - 35$

$= 3 \times 5 \times 9 - 35$

$= 15 \times 9 - 35$

$= 135 - 35$

$= 100$

or

$3 \times (5 \times 9) - 35 = 3 \times 45 - 35$

$= 135 - 35$

$= 100$

2. a. $16+8+2^3 = 16+8+8$
$= 2+8$
$= 10$

b. $6\times5+7^2 = 6\times5+49$
$= 30+49$
$= 79$

c. $36+3^2+12 = 36+9+12$
$= 4+12$
$= 16$

d. $4\times5^2-6^2 = 4\times25-36$
$= 100-36$
$= 64$

3. a. $8\times(7-2)^2+2 = 8\times5^2+2$
$= 8\times25+2$
$= 8\times12\frac{1}{2}$
$= 100$

b. $3^4+9+(5+4) = 81+9+(5+4)$
$= 81+9+9$
$= 9+9$
$= 18$

c. $\left(18-4^2\right)+2 = (18-16)+2$
$= 2+2$
$= 1$

d. $3^3+9+(5\times4) = 27+9+(5\times4)$
$= 27+9+20$
$= 3+20$
$= 23$

4. a. $\frac{5\left(5^2-5\right)}{2^2+1} = \frac{5(25-5)}{4+1}$
$= \frac{5\times20}{5}$
$= \frac{100}{5}$
$= 20$

b. $\frac{4\times5^2-6^2}{3^2-1} = \frac{4\times25-36}{9-1}$
$= \frac{100-36}{9-1}$
$= \frac{64}{8}$
$= 8$

c. $\dfrac{\left(4^2+2\right)^2}{(3+1)+2^3}=\dfrac{(16+2)^2}{(3+1)+8}$

$=\dfrac{8^2}{(3+1)+8}$

$=\dfrac{64}{(3+1)+8}$

$=\dfrac{64}{4+8}$

$=\dfrac{64}{12}$

$=5\dfrac{4}{12}$

$=5\dfrac{1}{3}$

d. $\dfrac{20-8+2^2}{3\times2^2-4}=\dfrac{20-8+4}{3\times4-4}$

$=\dfrac{20-8+4}{12-4}$

$=\dfrac{20-12}{12-4}$

$=\dfrac{8}{8}$

$=1$

5. a. $207\times1.3-1.51=269.1-1.51$

$=267.59$

b. $1.124+4+0.25=0.281+0.25$

$=0.531$

c. $14.1+7.9\times3.4-13.72=14.1+26.86-13.72$

$=40.96-13.72$

$=27.24$

6. a. $4\times50+(80+8)=4\times50+10$

$=4\times5$

$=20$

b. $49-9+3\times(24+6)\times4=49-9+3\times4\times4$

$=49-3\times4\times4$

$=49-12\times4$

$=49-48$

$=1$

c. $(7\times2)-(12+2)-(15+3)=14-6-5$

$=8-5$

$=3$

d. $\dfrac{22+11\times 6}{4+4+2}=\dfrac{2\times 6}{4+2}$

$=\dfrac{12}{4+2}$

$=\dfrac{12}{6}$

$=2$

Activity 10

Perform calculations involving integers.

1. a. $-24+(-6)=-30$

   b. $-32+(+8)=-24$

   c. $+32+(-6)=+26$

2. a. $-24-(+4)=-24+(-4)$

   $=-28$

   b. $+24-(-4)=+24+(+4)$

   $=+28$

   c. $-32-(+6)=-32+(-6)$

   $=-38$

3. a. $-32\times(-3)=+96$ b. $+32\times(-3)=-96$

   c. $+32\times(+4)=+128$

4. a. $-16+(-2)=+8$ b. $+16+(+8)=+2$

   c. $-16+(+4)=-4$

5. a. $-25+(+36)=+11$ b. $+25-(-36)=+61$

   c. $-40\times(+5)=-200$

6. a. $-40+(-5)=+8$ b. $+40\times(-3)=-120$

   c. $-8-(+2)=-10$

7. a. $-64-(-20)=-44$ b. $-64+(-20)=-84$

   c. $-64+(+2)=-32$

8. a. Cheque = \$40.00
Present balance = \$24.00

\$24.00 + (−\$40.00) = −\$16.00

Bill's account would be overdrawn by \$16.00.

b. Present location = 40 yard line
Loss = 8 yards

+40 + (−8) = +32

The ball should be placed at the 32 yard line.

c. Temperature at 6:00 a.m. = 6°C
Temperature drop = 20°C

6° + (−20°) = −14°

The temperature at 10:00 p.m. would be −14°C.

Activity 11

Perform calculations involving rational numbers.

1. a. $-\frac{7}{8}+\left(\frac{+3}{4}\right)=\frac{-7}{8}+\left(\frac{+6}{8}\right)$

$=\frac{-1}{8}$

b. $+\frac{3}{4}\times\left(-1\frac{1}{2}\right)=\frac{+3}{4}\times\left(\frac{-3}{2}\right)$

$=\frac{-9}{8}$

$=-1\frac{1}{8}$

c. $-6+\left(+1\frac{1}{4}\right)=-5\frac{4}{4}+\left(+1\frac{1}{4}\right)$

$=-4\frac{3}{4}$

2. a. $+\frac{7}{8}+\left(\frac{-3}{4}\right)=\frac{+7}{8}+\left(\frac{-6}{8}\right)$

$=\frac{+1}{8}$

b. $-\frac{3}{4}\times\left(+1\frac{1}{2}\right)=\frac{-3}{4}\times\left(\frac{+3}{2}\right)$

$=\frac{-9}{8}$

$=-1\frac{1}{8}$

c. $+6+\left(-1\frac{1}{4}\right)=+5\frac{4}{4}+\left(-1\frac{1}{4}\right)$

$=+4\frac{3}{4}$

3. a. $-6.2+(-5)=-11.2$

b. $+6.2-(-5)=+11.2$

c. $-4\times(+3.8)=-15.2$

4. a. $-4.2+(+2)=-2.1$

b. $+5.08\times(-0.2)=-1.016$

c. $6-(-2.2)=+8.2$

5. a. Pay cheque = \$65.00
Previous balance = $-$\$6.32$
$\$65.00+(-\$6.32)=+\$56.68$
Naomi's account now shows a balance of $+$\$56.68$.

b. Number of pies = 6

Number given away $=1\frac{1}{2}$

$$+6+\left(-1\frac{1}{2}\right)=+5\frac{2}{2}+\left(-1\frac{1}{2}\right)$$
$$=+4\frac{1}{2}$$

Edie has $4\frac{1}{2}$ pies left after giving $1\frac{1}{2}$ pies away.

Extra Help

1. a. $\frac{5}{6}+\frac{2}{3}=\frac{15+12}{18}$
$=\frac{27}{18}$
$=\frac{3}{2}$
$=1\frac{1}{2}$

b. $\frac{7}{9}+\frac{3}{4}=\frac{28+27}{36}$
$=\frac{55}{36}$
$=1\frac{19}{36}$

c. $11\frac{7}{8}-6\frac{5}{6}=(11-6)+\left(\frac{7}{8}-\frac{5}{6}\right)$
$=5+\frac{42-40}{48}$
$=5+\frac{2}{48}$
$=5\frac{1}{24}$

2. a. $\frac{5}{8}-\frac{7}{12}=\frac{5\times3}{8\times3}-\frac{7\times2}{12\times2}$

$=\frac{15}{24}-\frac{14}{24}$

$=\frac{1}{24}$

b. $9\frac{1}{8}-6\frac{7}{8}=(9-6)+\left(\frac{1}{8}-\frac{7}{8}\right)$

$=(9-6)+\left(\frac{-6}{8}\right)$

$=3+\left(\frac{-3}{4}\right)$

$=2\frac{1}{4}$

c. $10\frac{3}{8}-7\frac{1}{9}=(10-7)+\left(\frac{3}{8}-\frac{1}{9}\right)$

$=3+\frac{27-8}{72}$

$=3\frac{19}{72}$

or

$10\frac{3}{8}=10\frac{27}{72}$

$-7\frac{1}{9}=7\frac{8}{72}$

$3\frac{19}{72}$

3. a. $1\frac{1}{2}+2\frac{2}{3}+1\frac{1}{4}=(1+2+1)+\left(\frac{1}{2}+\frac{2}{3}+\frac{1}{4}\right)$

$=4+\left(\frac{6}{12}+\frac{8}{12}+\frac{3}{12}\right)$

$=4+\frac{17}{12}$

$=4+1+\frac{5}{12}$

$=5\frac{5}{12}$

b. $9\frac{1}{2}-11\frac{3}{4}+5\frac{1}{3}=(9-11+5)+\left(\frac{1}{2}-\frac{3}{4}+\frac{1}{3}\right)$

$=3+\left(\frac{6}{12}-\frac{9}{12}+\frac{4}{12}\right)$

$=3+\left(\frac{-3}{12}+\frac{4}{12}\right)$

$=3+\frac{1}{12}$

$=3\frac{1}{12}$

c. $7\frac{3}{4}-2\frac{1}{8}-3\frac{1}{3}=(7-2-3)+\left(\frac{3}{4}-\frac{1}{8}-\frac{1}{3}\right)$

$=2+\left(\frac{18}{24}-\frac{3}{24}-\frac{8}{24}\right)$

$=2+\left(\frac{15}{24}-\frac{8}{24}\right)$

$=2+\frac{7}{24}$

$=2\frac{7}{24}$

4. a. $\frac{2}{3}\times\frac{8}{16}\times\frac{9}{5}=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{16}}}\times\frac{\overset{3}{\cancel{9}}}{5}$

$=\frac{3}{5}$

Note: The 8 and 16 were cancelled first; then, the 2 from the 16 and the 2 from $\frac{2}{3}$ were cancelled next. Does it have to be done in this order? The answer is No. You could cancel the 2 and 16 first or the 3 and 9 first. The end result would be the same.

b. $\frac{5}{6}\times\frac{7}{15}\times\frac{18}{21}=\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}}\times\frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{15}}}\times\frac{\overset{1}{\cancel{18}}}{\underset{1}{\cancel{21}}}$

$=\frac{1}{3}$

c. $1\frac{1}{2}\times2\frac{2}{9}\times3\frac{3}{10}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{20}}}{\underset{1}{\cancel{9}}}\times\frac{\overset{11}{\cancel{33}}}{\underset{1}{\cancel{10}}}$

$=\frac{11}{1}$

$=11$

5. a. $1\frac{1}{3}-3\frac{1}{2}=\frac{4}{3}-\frac{7}{2}$ or $1\frac{1}{3}-3\frac{1}{2}=(1-3)+\left(\frac{1}{3}-\frac{1}{2}\right)$

$=\frac{8-21}{6}$ $=-2+\frac{2-3}{6}$

$=\frac{-13}{6}$ $=-2-\frac{1}{6}$

$=-2\frac{1}{6}$ $=-2\frac{1}{6}$

b. $5\frac{1}{4}+3\frac{1}{5}=(5+3)+\left(\frac{1}{4}+\frac{1}{5}\right)$ or $5\frac{1}{4}+3\frac{1}{5}=\frac{21}{4}+\frac{16}{5}$

$=8+\frac{5+4}{20}$ $=\frac{105+64}{20}$

$=8+\frac{9}{20}$ $=\frac{169}{20}$

$=8\frac{9}{20}$ $=8\frac{9}{20}$

c. $-1\frac{1}{8}+2\frac{1}{5}=(-1+2)+\left(-\frac{1}{8}+\frac{1}{5}\right)$ or $-1\frac{1}{8}+2\frac{1}{5}=\frac{-9}{8}+\frac{11}{5}$

$=1+\left(\frac{-5}{40}+\frac{8}{40}\right)$ $\quad=\frac{-45+88}{40}$

$=1+\frac{3}{40}$ $\quad=\frac{43}{40}$

$=1\frac{3}{40}$ $\quad=1\frac{3}{40}$

d. $-2\frac{5}{12}-3\frac{1}{7}=(-2-3)+\left(-\frac{5}{12}-\frac{1}{7}\right)$ or $-2\frac{5}{12}-3\frac{1}{7}=\frac{-29}{12}-\frac{22}{7}$

$=-5+\left(\frac{-35-12}{84}\right)$ $\quad=\frac{-203-264}{84}$

$=-5-\frac{47}{84}$ $\quad=\frac{-467}{84}$

$=-5\frac{47}{84}$ $\quad=-5\frac{47}{84}$

e. $-7\frac{1}{3}+3\frac{1}{7}=(-7+3)+\left(-\frac{1}{3}+\frac{1}{7}\right)$ or $-7\frac{1}{3}+3\frac{1}{7}=\frac{-22}{3}+\frac{22}{7}$

$=-4+\left(\frac{-7+3}{21}\right)$ $\quad=\frac{-154+66}{21}$

$=-4+\frac{-4}{21}$ $\quad=\frac{-88}{21}$

$=-4\frac{4}{21}$ $\quad=-4\frac{4}{21}$

f. $2\frac{5}{6}-1\frac{7}{8}=1+\left(\frac{40-42}{48}\right)$ or $2\frac{5}{6}-1\frac{7}{8}=\frac{17}{6}-\frac{15}{8}$

$=1-\frac{2}{48}$ $=\frac{68-45}{24}$

$=1-\frac{1}{24}$ $=\frac{23}{24}$

$=\frac{23}{24}$

Note: $1-\frac{1}{24}=\frac{24}{24}-\frac{1}{24}$

$=\frac{23}{24}$

6. a. $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}=\frac{1\times15}{2\times15}+\frac{1\times10}{3\times10}+\frac{1\times6}{5\times6}$

$=\frac{15}{30}+\frac{10}{30}+\frac{6}{30}$

$=\frac{15+10+6}{30}$

$=\frac{31}{30}$

$=1\frac{1}{30}$

b. $2\frac{1}{5}-3\frac{1}{4}+2\frac{1}{3}=(2-3+2)+\left(\frac{1}{5}-\frac{1}{4}+\frac{1}{3}\right)$ or $2\frac{1}{5}-3\frac{1}{4}+2\frac{1}{3}=\frac{11}{5}-\frac{13}{4}+\frac{7}{3}$

$=1+\frac{1\times12}{5\times12}-\frac{1\times15}{4\times15}+\frac{1\times20}{3\times20}$ $=\frac{132-195+140}{60}$

$=1+\frac{12}{60}-\frac{15}{60}+\frac{20}{60}$ $=\frac{77}{60}$

$=1+\frac{12-15+20}{60}$ $=1\frac{17}{60}$

$=1+\frac{-3+20}{60}$

$=1+\frac{17}{60}$

$=1\frac{17}{60}$

c. $5\frac{8}{9}+13\frac{1}{6}=(5+13)+\left(\frac{8}{9}+\frac{1}{6}\right)$ or $5\frac{8}{9}+13\frac{1}{6}=\frac{53}{9}+\frac{79}{6}$

$=18+\frac{48+9}{54}$ $=\frac{106+237}{18}$

$=18+\frac{57}{54}$ $=\frac{343}{18}$

$=18+\frac{19}{18}$ $=19\frac{1}{18}$

$=18+1\frac{1}{18}$

$=19\frac{1}{18}$

d. $3\frac{1}{17}-4\frac{2}{13}=(3-4)+\left(\frac{1}{17}-\frac{2}{13}\right)$ or $3\frac{1}{17}-4\frac{2}{13}=\frac{52}{17}-\frac{54}{13}$

$=-1+\frac{13-34}{221}$ $=\frac{676-918}{221}$

$=-1+\frac{-21}{221}$ $=\frac{-242}{221}$

$=-1\frac{21}{221}$ $=-1\frac{21}{221}$

Extensions

1. a. $1.86\times10=18.6$

   b. $343\times100=34\,300$

   c. $0.014\times100=1.4$

   d. $1000\times0.84=840$

   e. $0.0001\times10\,000=1$

2. a. $34.3\div100=0.343$

   b. $0.17\div10=0.017$

   c. $10.01\div1000=0.010\,01$

   d. $45\div1000=0.045$

   e. $0.0098\div10=0.000\,98$

Exploring Topic 2

Activity 1

Change a fraction to a percent and vice versa.

1. a. $\frac{1}{5}=\frac{1\times20}{5\times20}$

   $=\frac{20}{100}$

   $=20\%$

   $\therefore\frac{1}{5}=20\%$

   b. $\frac{3}{4}=\frac{3\times25}{4\times25}$

   $=\frac{75}{100}$

   $=75\%$

   $\therefore\frac{3}{4}=75\%$

   c. $\frac{4}{5}=\frac{4\times20}{5\times20}$

   $=\frac{80}{100}$

   $=80\%$

   $\therefore\frac{4}{5}=80\%$

2. a. $\frac{1}{3} = \frac{n}{100}$

$\frac{1\times100}{3} = \frac{n\times100}{100}$

$n = \frac{100}{3}$

$= 33\frac{1}{3}$

$\therefore \frac{1}{3} = 33\frac{1}{3}\%$

b. $\frac{5}{6} = \frac{n}{100}$

$\frac{5\times100}{6} = \frac{n\times100}{100}$

$n = \frac{500}{6}$

$= 83\frac{1}{3}$

$\therefore \frac{5}{6} = 83\frac{1}{3}\%$

c. $\frac{2}{15} = \frac{n}{100}$

$\frac{2\times100}{15} = \frac{n\times100}{100}$

$n = \frac{200}{15}$

$= 13\frac{1}{3}$

$\therefore \frac{2}{15} = 13\frac{1}{3}\%$

3. a. $\frac{4}{25} = \frac{4\times4}{25\times4}$

$= \frac{16}{100}$

$= 16\%$

$\therefore \frac{4}{25} = 16\%$

b. $\frac{19}{20} = \frac{19\times5}{20\times5}$

$= \frac{95}{100}$

$= 95\%$

$\therefore \frac{19}{20} = 95\%$

c. $\frac{5}{12} = \frac{n}{100}$

$\frac{5\times100}{12} = \frac{n\times100}{100}$

$n = \frac{500}{12}$

$= 41\frac{2}{3}$

$\therefore \frac{5}{12} = 41\frac{2}{3}\%$

4. a. $1\frac{1}{2} = \frac{3}{2} = \frac{n}{100}$

$\frac{3\times100}{2} = \frac{n\times100}{100}$

$n = \frac{300}{2}$

$= 150$

$\therefore 1\frac{1}{2} = 150\%$

b. $1\frac{1}{6} = \frac{7}{6} = \frac{n}{100}$

$\frac{7\times100}{6} = \frac{n\times100}{100}$

$n = \frac{700}{6}$

$= 116\frac{2}{3}$

$\therefore 1\frac{1}{16} = 116\frac{2}{3}\%$

c. $2\frac{2}{3} = \frac{8}{3} = \frac{n}{100}$

$$\frac{8 \times 100}{3} = \frac{n \times 100}{100}$$

$$n = \frac{800}{3}$$

$$= 266\frac{2}{3}$$

$$\therefore 2\frac{2}{3} = 266\frac{2}{3}\%$$

5. a. $\frac{1}{20} = \frac{1 \times 5}{20 \times 5}$

$$= \frac{5}{100}$$

$$= 5\%$$

$$\therefore \frac{1}{20} = 5\%$$

b. $1\frac{1}{50} = \frac{51}{50} = \frac{n}{100}$

$$\frac{51 \times 100}{50} = \frac{n \times 100}{100}$$

$$n = \frac{5100}{50}$$

$$= 102$$

$$\therefore 1\frac{1}{50} = 102\%$$

c. $\frac{17}{20} = \frac{17 \times 5}{20 \times 5}$

$$= \frac{85}{100}$$

$$= 85\%$$

$$\therefore \frac{17}{20} = 85\%$$

6. The statement 3 out of 20 means $\frac{3}{20}$.
As a percent,

$$\frac{3}{20} = \frac{3 \times 5}{20 \times 5}$$

$$= \frac{15}{100}$$

$$= 15\%$$

Therefore, 15% of the people were unemployed.

7. Class of 55 had 30 girls; therefore, the number of boys is $55 - 30 = 25$.

ratio of boys in class $= \frac{25}{55}$

$$\frac{25}{55} = \frac{n}{100}$$

$$\frac{25 \times 100}{55} = \frac{n \times 100}{100}$$

$$n = 45.\overline{45}$$

The percent of boys in the class was $45.\overline{45}\%$. When rounded to the nearest percent, the percent of boys is 45%.

8. a. $75\% = \frac{75}{100}$
$= \frac{3}{4}$

b. $35\% = \frac{35}{100}$
$= \frac{7}{20}$

c. $5\% = \frac{5}{100}$
$= \frac{1}{20}$

d. $12\frac{1}{2}\% = \frac{12\frac{1}{2} \times 2}{100 \times 2}$
$= \frac{25}{200}$
$= \frac{1}{8}$

e. $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$
$= 33\frac{1}{3} \div 100$
$= \frac{100}{3} \times \frac{1}{100}$
$= \frac{100}{300}$
$= \frac{1}{3}$

f. $16\frac{2}{3}\% = \frac{16\frac{2}{3}}{100}$
$= 16\frac{2}{3} \div 100$
$= \frac{50}{3} \times \frac{1}{100}$
$= \frac{50}{300}$
$= \frac{1}{6}$

Activity 2

Change a decimal to a percent and vice versa.

1. a. $0.6 = 60\%$
b. $0.04 = 4\%$
c. $0.32 = 32\%$
d. $0.1 = 10\%$

2. a. $0.125 = 12.5\%$
b. $0.009 = 0.9\%$
c. $0.105 = 10.5\%$
d. $0.0055 = 0.55\%$

3. a. $1.5 = 150\%$
b. $2.25 = 225\%$
c. $1.9 = 190\%$
d. $1.05 = 105\%$

4. a. $6\% = 0.06$
b. $19\% = 0.19$
c. $2\% = 0.02$
d. $100\% = 1.00$ or 1

5. a. $1\frac{1}{2}\% = 1.5\%$
$= 0.015$
b. $0.5\% = 0.005$

c. $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$

$= 33\frac{1}{3} \div 100$

$= \frac{100}{3} \times \frac{1}{100}$

$= \frac{100}{300}$

$= \frac{1}{3}$

$= 0.\overline{3}$

d. $275\% = \frac{275}{100}$

$= 2.75$

6. During the election, 0.85 of people voted. Therefore, during the election, 85% of people voted.

   During the election, (100 – 85)% of people did not vote. Therefore, during the election, 15% of people did not vote which means that they did not go to the polls.

7. There was 0.375 of a plane's cargo lost. Therefore, 37.5% of the plane's cargo was lost.

   There was (100 – 37.5)% of the cargo that made it to safety. Therefore, 62.5% of the cargo made it to safety.

Activity 3

Make calculations using percents.

1. $\frac{84}{25} = \frac{84 \times 4}{25 \times 4}$

$= \frac{336}{100}$

$= 336\%$

2. $x\%$ of $64 = 48$

$\frac{x}{100} = \frac{48}{64}$

$64x = 48 \times 100$

$\frac{64x}{64} = \frac{48 \times 100}{64}$

$x = \frac{3 \times 100}{4}$

$= \frac{300}{4}$

$= 75\%$

3. 95% of debt is \$760.00. Let x be the total debt.

$\frac{95}{100} = \frac{760.00}{x}$

$95x = 760.00 \times 100$

$\frac{95x}{95} = \frac{760 \times 100}{95}$

$x = \frac{76\,000.00}{95}$

$= \$800.00$

The total debt is \$800.00.

4. $\frac{45}{72} = \frac{n}{100}$

$72n = 45 \times 100$

$\frac{72n}{72} = \frac{45 \times 100}{72}$

$n = \frac{5 \times 100}{8}$

$= \frac{500}{8}$

$= 62.5\%$

5. 35% of \$105.00 is x.

$\frac{35}{100} = \frac{x}{105.00}$

$100x = 35 \times 105.00$

$\frac{100x}{100} = \frac{35 \times 105.00}{100}$

$x = 36.75$

Therefore, 35% of \$105.00 is \$36.75.

6. 30% of $x = 60$

$\frac{30}{100} = \frac{60}{x}$

$30x = 60 \times 100$

$\frac{30x}{30} = \frac{60 \times 100}{30}$

$x = 200$

Therefore, 30% of 200 is 60.

7. $5\% + 15\% + 27\% + 43\% = (5 + 15 + 27 + 43)\%$

$= (20 + 70)\%$

$= 90\%$

8. 75% of $55 = x$

$\frac{75}{100} = \frac{x}{55}$

$100x = 75 \times 55$

$\frac{100x}{100} = \frac{75 \times 55}{100}$

$x = 41.25$

Therefore, 75% of 55 is 41.25.

Activity 4

Solve problems involving percent.

1. Number of students in the class = 35
Number of boys in the class = 21
Number of girls in the class = $35 - 21 = 14$

$\frac{14}{35} = \frac{n}{100}$

$35n = 14 \times 100$

$\frac{35n}{35} = \frac{14 \times 100}{35}$

$n = \frac{1400}{35}$

$= 40$

Girls make up 40% of the class.

2. Number of problems in test = 90
Number of problems Milton got right = 72

$$\frac{72}{90} = \frac{n}{100}$$
$$90n = 72 \times 100$$
$$\frac{90n}{90} = \frac{72 \times 100}{90}$$
$$n = \frac{7200}{90}$$
$$= 80$$

Milton got 80% of the problems right.

3. Machine can put out 420 cases per day.
Machine is performing at 70% of capacity.
Let n be the number of cases actually produced.

$$\frac{n}{420} = \frac{70}{100}$$
$$100n = 70 \times 420$$
$$\frac{100n}{100} = \frac{70 \times 420}{100}$$
$$n = 294$$

The machine is producing 294 cases per day.

4. Number of girls polled = 240
Number of girls choosing coffee = 160
Number of girls choosing tea = $240 - 160 = 80$
Let the percent of girls choosing tea be n.

$$\frac{80}{240} = \frac{n}{100}$$
$$240n = 80 \times 100$$
$$\frac{240n}{240} = \frac{80 \times 100}{240}$$
$$n = \frac{800}{24}$$
$$= 33\frac{1}{3}$$

Of all the girls polled, $33\frac{1}{3}\%$ preferred tea.

5. Number of hamburgers sold in cafeteria = 240
Percent that were cheeseburgers = $66\frac{2}{3}\%$
Percent that were plain hamburgers = $100 - 66\frac{2}{3} = 33\frac{1}{3}\%$
Let the number of plain hamburgers be x.

$$\frac{x}{240} = \frac{33\frac{1}{3}}{100}$$
$$100x = 240 \times 33\frac{1}{3}$$
$$\frac{100x}{100} = \frac{240 \times 33\frac{1}{3}}{100}$$
$$x = \frac{8000}{100}$$
$$= 80$$

80 of the hamburgers sold were plain hamburgers.

6. Attendance at college on a certain day is 840 students.
Let x be the total enrolment. Remember the **total** goes on the bottom of the fraction.
840 is 80% of total enrolment.

$$\frac{840}{x} = \frac{80}{100}$$
$$80x = 840 \times 100$$
$$\frac{80x}{80} = \frac{840 \times 100}{80}$$
$$x = \frac{84\,000}{80}$$
$$= 1050$$

The total enrolment of the college is 1050 students.

7. Selling price of camper = \$16 900.00.
Rate of downpayment $= 12\frac{1}{2}\%$
Let n equal the amount of downpayment.
$12\frac{1}{2}\%$ of 16 900.00 is n.

$$\frac{12\frac{1}{2}}{100} = \frac{n}{16\,900.00}$$
$$100n = 12\frac{1}{2} \times 16\,900.00$$
$$\frac{100n}{100} = \frac{12\frac{1}{2} \times 16\,900.00}{100}$$
$$n = \frac{211\,250.00}{100}$$
$$= 2112.50$$

The downpayment to buy the camper was \$2112.50.

8. Alana bought a house for \$49 000.00.
Alana wants a profit of 9%.
Amount of profit would be 9% of 49 000.00.
Let the profit be n.

$$\frac{n}{49\,000.00} = \frac{9}{100}$$
$$100n = 9 \times 49\,000.00$$
$$\frac{100n}{100} = \frac{9 \times 49\,000.00}{100}$$
$$n = \frac{441\,000.00}{100}$$
$$= 4410.00$$

Alana should sell the house for
\$4410.00 + \$49 000.00 = \$53 410.00 to make a 9% profit.

Extra Help

1. a. $\frac{3}{8} = \frac{x}{100}$
$8x = 300$
$\frac{8x}{8} = \frac{300}{8}$
$x = 37\frac{1}{2}\%$ or 37.5%

b. $\frac{3}{4} = \frac{x}{100}$
$4x = 300$
$\frac{4x}{4} = \frac{300}{4}$
$x = 75\%$

c. $\frac{5}{6}=\frac{x}{100}$

$6x=500$

$\frac{6x}{6}=\frac{500}{6}$

$x=83\frac{1}{3}\%$ or $83.\overline{3}\%$

d. $\frac{13}{10}=\frac{x}{100}$

$10x=1300$

$\frac{10x}{10}=\frac{1300}{10}$

$x=130\%$

2. a. $\frac{1}{8}=0.125$

$=12.5\%$

b. $\frac{4}{3}=1.\overline{3}$

$=133.\overline{3}\%$

c. $\frac{9}{10}=0.9$

$=90\%$

d. $\frac{3}{5}=0.6$

$=60\%$

3. a. $25\%=\frac{25}{100}$ and $25\%=\frac{25}{100}$

$=\frac{1}{4}$ $=0.25$

b. $45\%=\frac{45}{100}$ and $45\%=\frac{45}{100}$

$=\frac{9}{20}$ $=0.45$

c. $116.25\%=\frac{116.25}{100}$ and $116.25\%=\frac{116.25}{100}$

$=\frac{11625}{10000}$ $=1.1625$

$=\frac{93}{80}$

$=1\frac{13}{80}$

d. $33\frac{1}{3}\%=\frac{33\frac{1}{3}}{100}$ and $33\frac{1}{3}\%=\frac{1}{3}$

$=\frac{\frac{100}{3}}{100}$ $\begin{array}{r}0.33\\3\overline{)1.00}\\9\\\hline 10\\9\\\hline 1\end{array}$

$=\frac{\frac{100}{3}\times 3}{100\times 3}$

$=\frac{100}{300}$ $33\frac{1}{3}\%=0.\overline{3}$

$=\frac{1}{3}$

e. $30\%=\frac{30}{100}$ and $30\%=\frac{30}{100}$

$=\frac{3}{10}$ $=0.3$

f. $16\frac{2}{3}\% = \frac{16\frac{2}{3}}{100}$ and $16\frac{2}{3}\% = \frac{1}{6}$

$= \frac{\frac{50}{3}}{100}$ $\quad 6\overline{)1.0000} = 0.1666$

$= \frac{\frac{50}{3} \times 3}{100 \times 3}$ $\quad 6$, 40, 36, 40, 36, 40, 36, 4

$= \frac{50}{300}$

$= \frac{1}{6}$

$16\frac{2}{3}\% = 0.1\overline{6}$

4. a. 40% of 75 = n

$\frac{40}{100} = \frac{n}{75}$

$100n = 40 \times 75$

$\frac{100n}{100} = \frac{3000}{100}$

$n = 30$

Therefore, 40% of 75 = 30.

b. 63% of 1156 = n

$\frac{63}{100} = \frac{n}{1156}$

$100n = 63 \times 1156$

$\frac{100n}{100} = \frac{72\,828}{100}$

$n = 728.28$

Therefore, 63% of 1156 = 728.28.

c. n% of 90 = 25

$\frac{n}{100} = \frac{25}{90}$

$90n = 100 \times 25$

$\frac{90n}{90} = \frac{2500}{90}$

$n = 27.\overline{7}$

Therefore, $27.\overline{7}$% of 90 = 25.

d. n% of 135 = 60

$\frac{n}{100} = \frac{60}{135}$

$135n = 100 \times 60$

$\frac{135n}{135} = \frac{6000}{135}$

$n = 44.\overline{4}$

Therefore, $44.\overline{4}$% of 135 = 60.

e. 17% of $n = 42$

$$\frac{17}{100} = \frac{42}{n}$$
$$17n = 42 \times 100$$
$$\frac{17n}{17} = \frac{4200}{17}$$
$$n \doteq 247.06 \quad \text{(to the nearest hundredth)}$$

Therefore, 17% of $247.06 \doteq 42$.

f. 39.5% of $n = 1034$

$$\frac{39.5}{100} = \frac{1034}{n}$$
$$39.5n = 100 \times 1034$$
$$\frac{39.5n}{39.5} = \frac{103\,400}{39.5}$$
$$n \doteq 2617.72 \quad \text{(to the nearest hundredth)}$$

Therefore, 39.5% of $2617.72 \doteq 1034$.

5. The fine would be $170 \text{ days} \times 5¢ / \text{day} = \$8.50$.
The cost of the book would be 85% of \$9.00

$$\frac{85}{100} = \frac{n}{9.00}$$
$$100n = 9.00 \times 85$$
$$\frac{100n}{100} = \frac{765.00}{100}$$
$$n = 7.65$$

The cost of the book would be \$7.65. To buy the book would be better since the cost would be \$0.85 less than the fine.

Extensions

1. a. Price of Car = \$8000.00
Selling Price = \$1000.00

$$\text{depreciation in 10} = \$8000.00 - \$1000.00$$
$$= \$7000.00$$

b. The amount of depreciation as a percent is calculated as follows.

$$\frac{7000}{8000} = \frac{x}{100}$$
$$8000x = 7000 \times 100$$
$$\frac{8000x}{8000} = \frac{700\,000}{8000}$$
$$x = 87.5$$

Therefore, the amount of depreciation is 87.5% of the new price.

2. a. The factory produces 9600 bottles in one day at 80% efficiency. Let the factory produce x bottles in one day at 100% efficiency.

$$\frac{80}{100} = \frac{9600}{x}$$
$$80x = 9600 \times 100$$
$$\frac{80x}{80} = \frac{960\,000}{80}$$
$$x = 12\,000$$

The factory will produce 12 000 bottles at a 100% efficiency level.

b. Using the previous information that the factory will produce 12 000 bottles at 100% efficiency level, let x be the number of bottles produced at a 60% efficiency level.

$$\frac{60}{100} = \frac{x}{12000}$$
$$100x = 60 \times 12\,000$$
$$\frac{100x}{100} = \frac{720\,000}{100}$$
$$x = 7200$$

Therefore, at a 60% efficiency level, the factory would produce 7200 bottles.

3. Lethal level of D.D.T. $= \frac{500}{1\,000\,000}$

a. Expressed as a decimal:

$$\frac{500}{1\,000\,000} = 0.000\,500 \text{ or } 0.0005$$

Notice the six zeroes in 1 000 000 and the six decimal places in 0.000 500.

b. Expressed as a percent:

$$0.000\,500 = 0.000\,500 \times 100$$
$$= 0.0500\% \text{ or } 0.05\%$$

4. a. 2.57% expressed as a mills is $2.57\% = \frac{2.57}{100}$

$$= \frac{25.7}{1000}$$

2.57% would be 25.7 mills.

b. 0.95% expressed as a mills is $0.95\% = \frac{0.95}{100}$

$$= \frac{9.5}{1000}$$

0.95% would be 9.5 mills.

c. 11.5% expressed as a mills is $11.5\% = \frac{11.5}{100}$

$$= \frac{115}{1000}$$

11.5% would be 115 mills.

5. a. 46.6 mills as a percent is $\frac{46.6}{1000} = \frac{4.66}{100}$

$= 4.66\%$

b. 2.59 mills as a percent is $\frac{2.59}{1000} = \frac{0.259}{100}$

$= 0.259\%$

c. 110.5 mills as a percent is $\frac{110.5}{1000} = \frac{11.05}{100}$

$= 11.05\%$

6. From the definition 1 mill $= \frac{1}{1000}$ of \$1.

a. $\$1.00 = 1.00 \times 1000$

$= 1000$ mills

b. $\$150.00 = 150 \times 1000$

$= 150\,000$ mills

c. $65¢ = \$0.65$

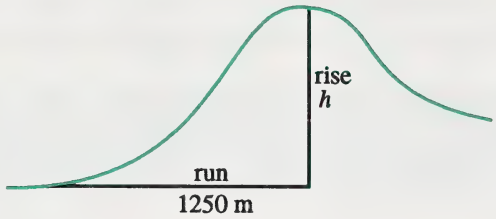
$= 0.65 \times 1000$

$= 650$ mills

7. Slope $= \frac{\text{rise}}{\text{run}}$

$= 7\%$ or $\frac{7}{100}$

Since the top of the hill is 1250 m away, the run is 1250 m and the rise is the height h of the hill.



Slope $= \frac{\text{rise}}{\text{run}}$

$\frac{7}{100} = \frac{h}{1250}$

$100h = 7 \times 1250$

$\frac{100h}{100} = \frac{8750}{100}$

$h = 87.5$

The hill has a height of 87.5 m.

8. a. Interest $= P \times r \times t$

$= 3000.00 \times 0.05 \times 4$

$= \$600.00$

The amount of interest would be \$600.00.

b. $I = prt$

$300.00 = P \times 0.05 \times 4$

$300.00 = 0.2P$

$\frac{0.2P}{0.2} = \frac{300}{0.2}$

$P = 1500.00$

$1500.00 must be invested to yield $300.00 in interest at 5% for 4 years.

9. A 50% chance of rain means that the likelihood of rain is the same as the likelihood of no rain.

10. Land tax went up 5%.
Last year's tax was $550.00.
Let the increase equal x dollars.

$\frac{x}{550} = \frac{5}{100}$

$100x = 5 \times 550$

$\frac{100x}{100} = \frac{2750}{100}$

$x = 27.50$

The increase is $27.50. Therefore, this year's land tax would be $550 + $27.50 = $577.50.

11. 15% of $1500.00 $= \frac{15}{100} \times \frac{1500.00}{1} = \$225.00$ or

$\frac{15}{100} = \frac{n}{1500.00}$

$n = \frac{15 \times 1500}{100}$

$= \$225.00$

If you double the rate and double the amount, the result will be a quadrupling effect.

Therefore, 30% of $3000.00 $= 4 \times 225.00$

$= \$900.00$

Exploring Topic 3

Activity 1

Find the square of a number.

1. The area represents the square of 36. Therefore, the square drawn would cover $36^2 = 1296$ grid squares.

2. $16^2 = 16 \times 16$

$= 256$

3. $(-21)^2 = (-21)\times(-21)$
$= 441$

4. a. Give the square of 3.15.

Enter	Display
3.15	3.15
x^2	9.9225

or

Enter	Display
3.15	3.15
×	3.15
=	9.9225

or

Enter	Display
3.15	3.15
×	3.15
3.15	3.15
=	9.9225

The enter sequence may have to be 3.15, Inv , $\sqrt{}$.

5. $38^2 = 38\times 38$
$= 1444$

Enter	Display
38	38
x^2	1444

or

Enter	Display
38	38
×	38
=	1444

or

Enter	Display
38	38
×	38
38	38
=	1444

The enter sequence may have to be 38, Inv , $\sqrt{}$.

6. $(3.5)^2 = 3.5 \times 3.5$

$= 12.25$

7. The tiles can be thought of as grid squares. The number of tiles is equal to 11^2. The square of 11 is $11 \times 11 = 121$. Mr. Abdul will need 121 tiles.

8. The area of the wall is $(2.47\text{ m})^2 = 2.47\text{ m} \times 2.47\text{ m}$

$= 6.1009\text{ m}^2$

This is greater than 6 m^2. Noel will not have enough wallpaper to finish the project.

9. $A = s^2$

$A = s \times s$

$A = 8\text{ cm} \times 8\text{ cm}$

$A = 64\text{ cm}^2$

The area of the square is 64 cm^2.

10. $A = \pi r^2$

$A = 3.14 \times 15^2$

$A = 3.14 \times 225$

$A = 706.5\text{ cm}^2$

$A \doteq 707\text{ cm}^2$ (rounded to the nearest cm^2)

The area of the circle is approximately 707 cm^2.

Activity 2

Find the square root of a number.

1. a. $\sqrt{121} = 11$

Enter	Display
121	121
$\sqrt{}$	11

b. Can you recognize 81 as a perfect square?

Since $81 = 9 \times 9$, then $\sqrt{81} = 9$.

Enter	Display
81	81
$\sqrt{}$	9

c. Here again, 49 can be recognized as a perfect square.

Since $49 = 7 \times 7$, then $\sqrt{49} = 7$.

Enter	Display
49	49
$\sqrt{}$	7

d. $\sqrt{196} = 14$

Enter	Display
196	196
$\sqrt{}$	14

e. $\sqrt{6889} = 83$

Enter	Display
6889	6889
$\sqrt{}$	83

f. $\sqrt{5^2 - 3^2} = \sqrt{25 - 9}$

$= \sqrt{16}$

$= 4$

Enter	Display
5	5
$\times$	5
$=$	25
$-$	25
3	3
$\times$	3
$=$	16
$\sqrt{}$	4

2. a.

Enter	Display
10	10
$\sqrt{}$	3.16227766

Therefore, $\sqrt{10}$ is about 3.162 rounded to three decimal places.

b. $\sqrt{40}$

Enter	Display
40	40
$\sqrt{}$	6.32455532

Therefore, $\sqrt{40}$ is about 6.325 rounded to three decimal places.

c. $\sqrt{56}$

Enter	Display
56	56
$\sqrt{}$	7.483314774

Therefore, $\sqrt{56}$ is about 7.483 rounded to three decimal places.

d. $\sqrt{255}$

Enter	Display
255	255
$\sqrt{}$	15.96871942

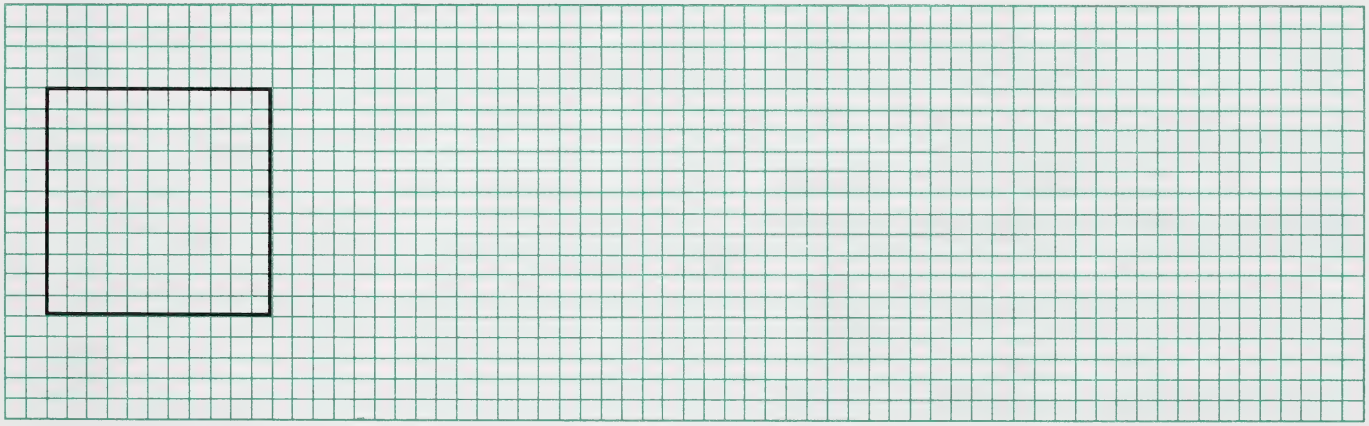
Therefore, $\sqrt{255}$ is about 15.969 rounded to three decimal places.

e. $\sqrt{6401}$

Enter	Display
6401	6401
$\sqrt{}$	80.00624976

Therefore, $\sqrt{6401}$ is about 80.006 rounded to three decimal places.

3.



The side length of this square is 11 units and its area is 121 grid squares. You can also think of the side length to be the square root of 121. $121 = 11 \times 11$ so $\sqrt{121} = 11$. The length of each side of the square is 11 units.

4. The side length of the field is $\sqrt{1155\ \text{m}^2} \doteq 34\ \text{m}$ rounded to the nearest metre. There are four sides; so the perimeter is $4 \times 34\ \text{m} \doteq 136\ \text{m}$ to the nearest metre. The length of fence required to enclose the field is 136 metres.

5. $A = \pi r^2$

$314 = 3.14r^2$

$\frac{314}{3.14} = \frac{3.14r^2}{3.14}$

$r^2 = 100$

$r = \sqrt{100}$

$= 10$

The radius of the circle is 10 cm.

6. a. $\sqrt{9} < \sqrt{11} < \sqrt{16}$

$3 < \sqrt{11} < 4$

11 is about $\frac{2}{7}$ of the way from 9 to 16.

To find $\frac{2}{7}$ use these calculations.

$11 - 9 = 2$

$16 - 9 = 7$

$\frac{2}{7} \doteq 0.285\,714\,285$

$\doteq 0.3$ (rounded to one decimal place)

$\sqrt{11} \doteq 3 + \frac{1}{3}$

$\doteq 3.3$ (rounded to one decimal place)

b. $\sqrt{36} < \sqrt{38} < \sqrt{49}$

$6 < \sqrt{38} < 7$

38 is about $\frac{2}{13}$ of the way from 36 to 49.

To find $\frac{2}{13}$ use these calculations.

$38 - 36 = 2$

$49 - 36 = 13$

$\frac{2}{13} \doteq 0.153\,846\,153$

$\doteq 0.2$ (rounded to one decimal place)

$\sqrt{38} \doteq 6 + 0.2$

$\doteq 6.2$ (rounded to one decimal place)

c. $\sqrt{81} < \sqrt{95} < \sqrt{100}$

$9 < \sqrt{95} < 10$

95 is about $\frac{14}{19}$ of the way from 81 to 100.

To find $\frac{14}{19}$ use these calculations.

$95 - 81 = 14$

$100 - 81 = 19$

$\frac{14}{19} \doteq 0.736\,842\,105$

$\doteq 0.7$ (rounded to one decimal place)

$\sqrt{95} \doteq 9 + 0.7$

$\doteq 9.7$ (rounded to one decimal place)

Extra Help

1. The length of each side is approximately 5.1 cm.
   Therefore, $\sqrt{26} \doteq 5.1$ rounded to one decimal place.

2. The length of each side is approximately 5.9 cm.
   Therefore, $\sqrt{35} \doteq 5.9$ rounded to one decimal place.

3. The length of each side is approximately 9.9 cm.
   Therefore, $\sqrt{98} \doteq 9.9$ rounded to one decimal place.

4. The length of each side is approximately 5.4 cm.
   Therefore, $\sqrt{29} \doteq 5.4$ rounded to one decimal place.

Extensions

Your answer need only be a number.

1. $15^2 = 225$
   $(1 \times 2 = \mathbf{2})$

2. $35^2 = 1225$
   $(3 \times 4 = \mathbf{12})$

3. $55^2 = 3025$
   $(5 \times 6 = \mathbf{30})$

4. $65^2 = 4225$
   $(6 \times 7 = \mathbf{42})$

5. $85^2 = 7225$
   $(8 \times 9 = \mathbf{72})$

6. $95^2 = 9025$
   $(9 \times 10 = \mathbf{90})$

7. $1005^2 = 1010025$
   $(100 \times 101 = \mathbf{10100})$

Exploring Topic 4

Activity 1

Evaluate powers such as 5^6, 94^2, $(-11)^3$, and $(-3)^5$.

1. a. $23 \times 23 \times 23 \times 23 = 23^4$ since 23 occurs as a factor four times.

   b. $(-6) \times (-6) \times (-6) \times (-6) = (-6)^4$ since -6 occurs as a factor four times.

   c. $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^6$ since 8 occurs as a factor six times.

2. a. $5^4 = 5 \times 5 \times 5 \times 5$
   $= 625$

   b. $7^2 = 7 \times 7$
   $= 49$

c. 12^3

Enter	Display
12	12
x^y or y^x	12
3	3
=	1728

$\therefore 12^3 = 1728$

d. 2^6

Enter	Display
2	2
x^y or y^x	2
6	6
=	64

$\therefore 2^6 = 64$

e. $3^4 = 3\times3\times3\times3$
$= 81$

3. a. $(-4)^3 = (-4)\times(-4)\times(-4)$
$= -64$

b. $-2^4 = -(2\times2\times2\times2)$
$= -16$

c. $-(-3)^5 = -[(-3)\times(-3)\times(-3)\times(-3)\times(-3)]$
$= -(-243)$
$= 243$

Remember that a negative times a negative equals a positive and $-1\times-243 = 243$.

d. 8^3

Enter	Display
8	8
x^y or y^x	8
3	3
=	512

$\therefore 8^3 = 512$

e. $(-10)^6$

Enter	Display
10	10
+/-	-10
x^y or y^x	-10
6	6
=	1000000

$\therefore (-10)^6 = 1\,000\,000$

4. a. 24^5

Enter	Display
24	24
x^y or y^x	24
5	5
=	7962624

$\therefore 24^5 = 7\ 962\ 624$

b. 64^4

Enter	Display
64	64
x^y or y^x	64
4	4
=	16777216

$\therefore 64^4 = 16\ 777\ 216$

c. 8^8

Enter	Display
8	8
y^x	8
8	8
=	16777216

$\therefore 8^8 = 16\ 777\ 216$

Did you notice that the answers for b and c are the same? In other words $64^4 = 8^8$.

d. 2^{21}

Enter	Display
2	2
y^x	2
21	21
=	2097152

$\therefore 2^{21} = 2\ 097\ 152$

e. 549^1

Enter	Display
549	549
y^x	549
1	1
=	549

$\therefore 549^1 = 549$

Try different bases to the power of 1. You should find that any base to the power of 1 gives you the chosen base as your answer.

5. In thirty days, the fruit fly population would be 10×4^6. This means that in thirty days there would be $10 \times 4096 = 40\,960$ fruit flies.

Check:

Number of days	Population
0	10
5	40
10	160
15	640
20	2 560
25	10 240
30	40 960

6. The population becomes $10\left(2^{15}\right) = 10 \times 32\,768$
$= 327\,680.$

After five hours there would be 327 680 bacteria in the test tube.

Check:

Number of periods		Population
0		10
1 h	1	20
	2	40
	3	80
2 h	4	160
	5	320
	6	640
3 h	7	1 280
	8	2 560
	9	5 120
4 h	10	10 240
	11	20 480
	12	40 960
5 h	13	81 920
	14	163 840
	15	327 680

Activity 2

Use the Product Law to simplify expressions such as $(4^8)(4^3)$ and $(5^4)(5^{13})$.

1. $9^{10} \times 9^{20} = 9^{10+20}$
   $= 9^{30}$

2. $(-4) \times (-4)^{20} = (-4)^{1+20}$
   $= (-4)^{21}$

3. $41^{17} \times 41^{26} = 41^{17+26}$
   $= 41^{43}$

4. $120^2 \times 120^{30} = 120^{2+30}$
   $= 120^{32}$

5. $(-32)^7 \times (-32)^7 = (-32)^{7+7}$
   $= (-32)^{14}$

6. $21^{16} \times 21^2 = 21^{16+2}$
   $= 21^{18}$

Activity 3

Use the Quotient Law to simplify expressions such as $\frac{7^5}{7^3}$ and $\frac{2^{12}}{2^{11}}$.

1. a. $9^{10} \div 9^9 = 9^{10-9}$
   $= 9^1$

   b. $10^{27} \div 10^4 = 10^{27-4}$
   $= 10^{23}$

   c. $14^{12} \div 14 = 14^{12-1}$
   $= 14^{11}$

   d. $\frac{272^{12}}{272^{11}} = 272^{12-11}$
   $= 272^1$

   e. $\frac{3^{60}}{3^{30}} = 3^{60-30}$
   $= 3^{30}$

   f. $11^{15} \div 11^5 = 11^{15-5}$
   $= 11^{10}$

2. a. $314^7 \div 314^6 = 314^{7-6}$
   $= 314^1$
   $= 314$

   b. $606^{112} \div 606^{111} = 606^{112-111}$
   $= 606^1$
   $= 606$

c. $8^{43} + 8^{40} = 8^{43-40}$

$= 8^3$

$= 512$

Activity 4

Evaluate powers with zero and negative exponents such as 4^0, 7^0, and 6^{-1}.

1. a. $(-5)^0 = 1$

b. $7^0 = 1$

c. $125^0 = 1$

d. $3^{-3} = \frac{1}{3^3}$

$= \frac{1}{27}$

$\doteq 0.037$

Enter	Display
3	3
x^y or y^x	3
3	3
=	27
Inv or $2^{nd}F$	27
$1/x$	0.037037037

Rounded to three decimal places, $3^{-3} = 0.037$.

e.

Enter	Display
5	5
y^x	5
2	2
+/-	−2
=	0.04

$5^{-2} = \frac{1}{5^2}$ or $5^{-2} = 0.04$

$= \frac{1}{25}$

f.

Enter	Display
10	10
y^x	10
5	5
+/-	−5
=	0.00001

$10^{-5} = \frac{1}{10^5}$ or $10^{-5} = 0.000\,01$

$= \frac{1}{10\,000}$

2. a. $9^{-12} = \frac{1}{9^{12}}$

b. $54^{-3} = \frac{1}{54^{3}}$

3. a. $2^{-1} + 3^{-1} = \frac{1}{2} + \frac{1}{3}$ where $\frac{1}{3} = \frac{2}{6}$

$= \frac{5}{6}$ $+ \frac{1}{2} = \frac{3}{6}$

$\frac{5}{6}$

b. $5^{0} + 2^{-2} + 5^{-1} = 1 + \frac{1}{2^{2}} + \frac{1}{5^{1}}$ where $1 = 1$

$= 1 + \frac{1}{4} + \frac{1}{5}$ $\frac{1}{4} = \frac{5}{20}$

$= 1\frac{9}{20}$ $+ \frac{1}{5} = \frac{4}{20}$

$1\frac{9}{20}$

c. $\frac{1}{\left(2^{-1} + 4^{-1}\right)} = \frac{1}{\left(\frac{1}{2} + \frac{1}{4}\right)}$

$= \frac{1}{\left(\frac{2}{4} + \frac{1}{4}\right)}$

$= \frac{1}{\frac{3}{4}}$

$= 1 \div \frac{3}{4}$

$= 1 \times \frac{4}{3}$

$= \frac{4}{3}$

$= 1\frac{1}{3}$

d. $\frac{2^{2} \times 3^{2}}{3^{3}} = \frac{4 \times 9}{27}$ or $\frac{2^{2} \times 3^{2}}{3^{3}} = 2^{2} \times 3^{2-3}$

$= \frac{36}{27}$ $= 2^{2} \times 3^{-1}$

$= 1\frac{9}{27}$ $= 4 \times \frac{1}{3}$

$= 1\frac{1}{3}$ $= \frac{4}{3}$

$= 1\frac{1}{3}$

e. $(5^3+5^5)\times 2^2 = 5^{3-5}\times 2^2$

$= 5^{-2}\times 2^2$

$= \frac{1}{5^2}\times\frac{2^2}{1}$

$= \frac{1}{25}\times 4$

$= \frac{4}{25}$

or

$(5^3+5^5)\times 2^2 = (125+3125)\times 4$

$= 0.04\times 4$

$= 0.16$

f. $25^{15}\times 25^{-15} = 25^{15+(-15)}$

$= 25^0$

$= 1$

Extra Help

1. $2^{-2} = \frac{1}{2^2}$

$= \frac{1}{4}$

2. $3^0 = 1$

3. $15^0 = 1$

4. $1^0 = 1$

5. $3^{-1} = \frac{1}{3^1}$

$= \frac{1}{3}$

6. $5^{-1} = \frac{1}{5^1}$

$= \frac{1}{5}$

7. $2^{-3} = \frac{1}{2^3}$

$= \frac{1}{8}$

8. $10^{-4} = \frac{1}{10^4}$

$= \frac{1}{10\,000}$ or 0.0001

9. $1000^0 = 1$

Extensions

1. a. $(10^3)^5 = 10^{3\times 5}$

$= 10^{15}$

b. $(2^9)^2 = 2^{9\times 2}$

$= 2^{18}$

c. $(11^5)^2 = 11^{5\times 2}$

$= 11^{10}$

d. $(5^2)^{-3} = 5^{2\times(-3)}$

$= 5^{-6}$

2. a. $(5^3)^2 = 5^{3\times 2}$

$= 5^6$

$= 15\,625$

b. $(10^{-1})^2 = 10^{-2}$

$= \frac{1}{10\times 10}$

$= \frac{1}{100}$ or 0.01

c. $\left[(-2)^2\right]^3 = (-2)^{2\times 3}$

$= (-2)^6$

$= (-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)$

$= 64$

d. $\left(3^{-2}\right)^{-1} = 3^{-2\times(-1)}$

$= 3^2$

$= 3\times 3$

$= 9$

3. a. $(1.05)^5$

Enter	Display
1.05	1.05
x^y or y^x	1.05
5	5
=	1.276281562

Therefore, $(1.05)^5 = 1.276$ rounded to three decimal places.

b. $\left(\frac{2}{5}\right)^3$

Enter	Display
2	2
÷	2
5	5
=	0.4
y^x	0.4
3	3
=	0.064

$\therefore \left(\frac{2}{5}\right)^3 = 0.064$

c. $(3.14)^{21} \div (3.14)^{20} = (3.14)^{21-20}$

$= 3.14^1$

$= 3.14$

d. $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^5$

$= 0.010\,24$

$\doteq 0.010$ (rounded to three decimal places)

Enter	Display
2	2
+	2
5	5
=	0.4
x^y or y^x	0.4
5	5
=	0.01024

$\therefore \left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 \doteq 0.010$

e. $(0.005)^{-2}$

Enter	Display
0.005	0.005
x^y or y^x	0.005
2	2
+/-	−2
=	40000

$\therefore (0.005)^{-2} = 40\,000$

f. $(0.125)^{-5}$

Enter	Display
0.125	0.125
x^y or y^x	0.125
5	5
+/-	−5
=	32768

$\therefore (0.125)^{-5} = 32\,768$

g. $\left(\frac{2}{3}\right)^0 = 1$

h. $\left(\frac{9}{16}\right)^{-3}$

Enter	Display
9	9
[+]	9
16	16
[=]	0.5625
[x^y] or [y^x]	0.5625
3	3
[+/-]	−3
[=]	5.618655692

Therefore, $\left(\frac{9}{16}\right)^{-3} = 5.619$ rounded to three decimal places.

4. a. $\left(-\frac{2}{3}\right)^9 \times \left(-\frac{2}{3}\right)^9 = \left(-\frac{2}{3}\right)^{9+9}$

$= \left(-\frac{2}{3}\right)^{18}$

b. $\left(\frac{1}{2}\right)^{14} \times \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{14+3}$

$= \left(\frac{1}{2}\right)^{17}$

c. $\left(\frac{1}{2}\right)^{14} \div \frac{1}{2} = \left(\frac{1}{2}\right)^{14-1}$

$= \left(\frac{1}{2}\right)^{13}$

d. $(1.01)^{15} \div (1.01)^5 = (1.01)^{15-5}$

$= (1.01)^{10}$

e. $\left(\frac{3}{11}\right)^4 \div \left(\frac{3}{11}\right)^6 = \left(\frac{3}{11}\right)^{4-6}$

$= \left(\frac{3}{11}\right)^{-2}$

5. a. $\left(\frac{7}{10}\right)^{-5} = \frac{1}{\left(\frac{7}{10}\right)^5}$ or $\left(\frac{10}{7}\right)^5$ b. $(12.75)^{-9} = \frac{1}{(12.75)^9}$

6. $(1.1)^5 = 1.61051$. To the nearest cent, Nadia's savings after five years will be $\$100.00 \times 1.6105 = \$161.05$. She would earn $\$161.05 - \$100.00 = \$61.05$ in interest in five years.

7. To the nearest cent, Simon's savings in eight years will grow to

$600.00 \times (1.1)^8 = \$600.00 \times 2.14358881$

$= 1286.153286$

$= \$1286.15$ (rounded to the nearest cent)

Simon would earn $\$1286.15 - \$600.00 = \$686.15$ in interest in eight years.

Mathematics 13

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